UNIT I

BASICS OF MECHANISMS

SYLLUBUS:

"Terminology and Definitions-Degree of Freedom Mobility-Kutzbach criterion-Grashoff's law-Kinematics Inversions of 4-bar chain and slider crank chains-Mechanical Advantage-Transmission angle-Description of common Mechanisms-Single, double and offset slider mechanisms - Quick return mechanisms - Ratchets and escapements - Indexing Mechanisms - Rocking Mechanisms - Straight line generators-Design of Crank-rocker Mechanisms".

TERMINOLOGY AND DEFINITIONS

Machine:

- It is a device, which takes in available energy and converts it into useful work.

Example: Shaper, Lathe, Cutting of threads, turning a rod.

Mechanics (Theory) of Machines:

- Branch of engineering which deals with the relative motion and forces between various machine elements;

Types:

1. Kinematics of Machines:

- Deals with relative motion without considering the forces.

2. Dynamics of Machines:

- Deals with the forces and the effect of forces on machine components when they act on them.

3. Kinetics of Machines:

- Deals with the forces, which are formed due to the combined action mass and motion of machine elements.

4. Static:

- Deals with the forces and its effects one machine parts while the latter is at rest.

Simple Mechanism:

- Resistant Body: A body is said to be a resistant body if it is able to transmit the forces with least possible deformation. Example: Springs, belts, oils in hydraulic press.

Link or Element:

- Each part of a machine that moves relative to some other part is known as a link.

Characteristics of Link:

(a) Should have relative motion

(b) Must be a resistant body.

Types of Links:

- Rigid: It undergoes no deformation; Example: crank, connecting rod.
- Flexible: Partial deformation; Example: springs, belts, ropes.
- Fluid: Motion is transmitted by this link by deformation.

Kinematic pairs:

It has two elements (or) links together which have relative motion between them.

Classification is based on:

I. Types of Constant Lower Pair

- Higher pair

II. Types of Constraint Closed pair

- Unclosed pair

III. Types of relative motion Sliding pair

- Turning pair
- Rolling pair
- Screw pair
- Spherical pair

Lower Pair:

If a pair motion has surface contact between the elements. Example:

- Piston reciprocating in a cylinders
- Shaft rotates in a bearing. (Note: Contacting surfaces are similar)

Higher Pair:

In higher pair there is a line or point contact between the elements.

Example: Cam and follower. (Note: Contact surfaces are different.)

Closed Pair:

In this pair, two elements are held together mechanically; Example: All lower pair

Unclosed Pair:

The two elements are not held together mechanically; Example: Cam and followers.

Sliding Pair:

The two elements have a sliding motion relative to each other. Example: Piston and cylinder pair rectangular rod is rectangular line.

Turing Pair:

- When the two elements are connected such that the element revolves about the other element.
- Example: Shaft rotates in the bearing rotation of a crank in a slider crank mechanism.

Rolling pair:

When one element is free to roll on another element. Example: The belt and pulley surfaces constitute rolling pair.

Screw Pair:

- In this type the contacting surface is having threads. It is also called a helical pair one element turns about another element by means of thread only.
- Example: A bolt and nut arrangement screw jack for lifting heavy weights.

Spherical Pair:

One element is in the form of sphere and turns about the fixed element; Example: ball and socket joint

Constrained Motions:

Constraint means: Limitation of motion (or) action.

- Completely Constraint: Moves in a definite direction

  Example: square bar moving in square hole.
- Incompletely Constraint: Moves in all direction (1) direction.

  Example: Circular bar moving in a circular hole.
- Successfully Constraint: Motion is not completed by itself but by some other means.

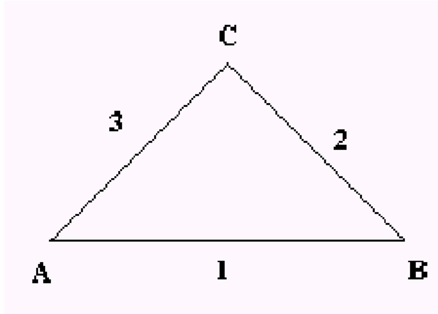
Kinematic Chain:

Kinematic pairs are completed in such a way that a last link is joined to the first link to transmit definite motion (constrained motion) is called as Kinematic Chain:

L = 2 p - 4 …. (1)

L = 2 / 3 (J + 2)... (2)

L = No of Links P no of pairs



J = No of Joints

Example:

From the diagram

L = 3, P = 3, J = 3,

Substituting L, P in (1), (2)

Since the values of L, J not satisfied ABC does not form a Kinematic chain but forms a structure.

Difference between Machine and Structure:

Machine

- Has relative motion between its members
- Transforms available energy into possible work.
- Members are meant to transmit Motion and force, **Example**: Shaper, Lathe

Structure

- Has no relative motion
- does not transforms so
- Members are meant to accept the loads.
- Example: Bridge

Types of Joints:

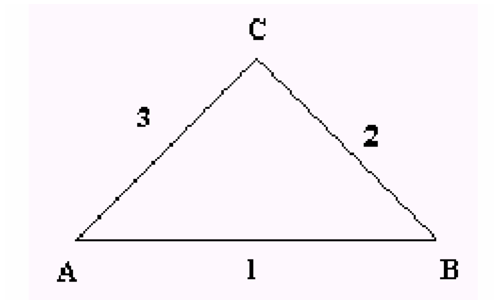
(a) Binary Joint: If two links are connected at the same end it is called as binary joint.

(b) A.W Klein:

$$J + h/2 = 3/2\, n - 2$$

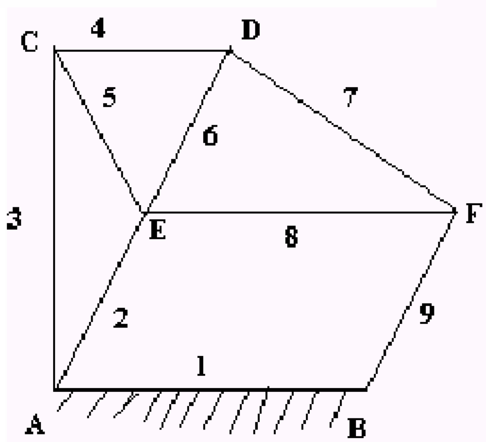
J - Joints (B); h - higher pairs; n - links

The diagram is a Kinematic Chain.



(d) Quartenary Joint:

A joint with four link is a quartenary joint. It is equal to four binary joint.



$$\frac{\text{Binary joint}}{B}$$

$$\frac{\text{Ternary Joint}}{A, C, D, F}$$

$$\frac{\text{Quarternary Joint}}{E}$$

J = 13

n = 9

$$J + h/2 = 3/2\ n - 2$$

$$13 + 0 = 3/2\,(9) - 2$$

$$13 = 11.5$$

It is not a kinematic chain but forms a structure.

Mechanism:

- When one link of Kinematic chain is fixed. It is known as mechanism. It transforms or is transmitting the motion.

  Example: engine, indicator, type writer.

Difference between Machine and Mechanism:

Machine	Mechanism
1. It is like the human body, it transforms energy into useful work.,	1. It is like frame work and has definite motion between various links.
2. It relates to energy only.	2. Relates to motion
3. It has many links.	3. It also has many links.
4. E.g. lathe, shaper	4. E.g. Engine ,indicator, typewriter

Mechanism

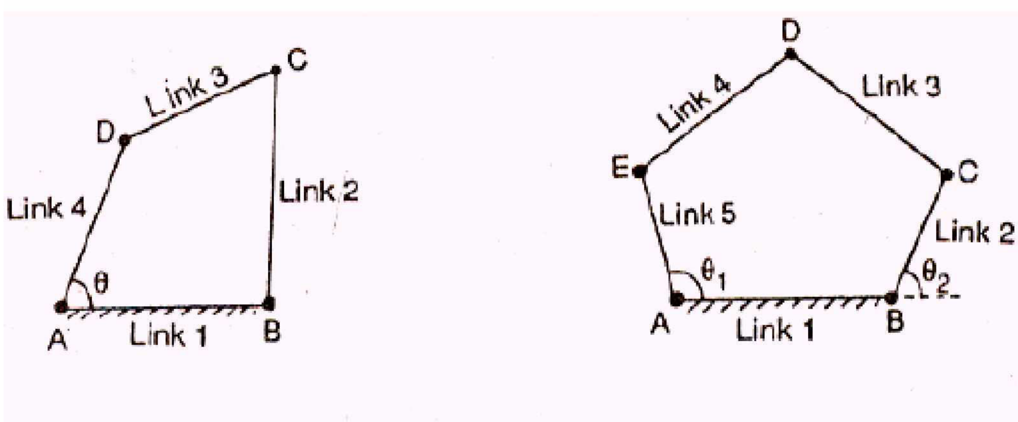
SIMPLE | COMPOUND

A mechanism with four links is called simple mechanism.

A mechanism with more than four links is called compound mechanism.

Degree of Freedom for plane Mechanism m (mobility):

It is defined as the no of input motions, which must be independently controlled in order to bring mechanism into useful engineering purpose.



(*a*) Four bar chain. **m=1** (*b*) Five bar chain.**m=2**

Kutzbach eriterion:

l = no of links

1 is fixed (l - 1) movable link 3 (l -1) Number of degree of freedom before it is connected to any other link. In general l number of links is connected by number of binary joints (or) lower pairs and h number of higher pairs, then the number of degrees of freedom of a mechanism is

$n = 3 (l - 1) – 2j– h.$

$n = 3 (l - 1) - 2j - h$ (Kutzbach criterion)

Grubler's criterion for plane motion:

$n = 3 (l - 1) - 2 j - h$

When $h = 0$, $n = 1$

We get a constrained motion given by

$3 l - 2 j - 4 = 0$

Grashof's Law:

- v The sum of the longest and the shortest length should not be greater than the sum of remaining two links length if there is to be continuous relative motion between the two links.
- v In a four-bar linkage, we refer to the *line segment between hinges* on a given link as a **bar** where:
- *s* = length of shortest bar
- *s* = length of longest bar
- *p, q* = lengths of intermediate bar

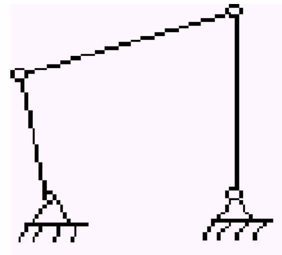
Grashof's theorem states that a four-bar mechanism has *at least* one revolving link if

$s + l <= p + q$ (1) and all three mobile links will rock if

$s + l > p + q$ (2)

The inequality 1 is **Grashof's criterion**.

- v The link opposite the frame is called the **coupler link**, and the links which are hinged to the frame are called **side links**.
- v A link which is free to rotate through 360 degree with respect to a second link will be said to **revolve** relative to the second link (not necessarily a frame).
- v If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.



Some important concepts in link mechanisms are:

- **Crank**: A side link which revolves relative to the frame is called a *crank*.
- **Rocker**: Any link which does not revolve is called a *rocker*.
- **Crank-rocker mechanism**: In a four bar linkage, if the shorter side link revolves and the other one rocks (*i.e.*, oscillates), it is called a *crank-rocker mechanism*.
- **Double-crank mechanism**: In a four bar linkage, if both of the side links revolve, it is called a *double-crank mechanism*.
- **Double-rocker mechanism**: In a four bar linkage, if both of the side links rock, it is called a *double-rocker mechanism*

All four-bar mechanisms fall into one of the four categories listed in Table

Case	l + s vers. p + q	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Doubl rocker
4	=	Any	Change point
5	>	Any	Double-rocker

Table Classification of Four-Bar Mechanisms

- From table, we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links.
- However, this condition is necessary but not sufficient. Mechanisms satisfying this condition fall into the following three categories:
- When the shortest link is a side link, the mechanism is a crank-rocker mechanism. The shortest link is the crank in the mechanism.
- 2. When the shortest link is the frame of the mechanism, the mechanism is a double crank mechanism.

- 3. When the shortest link is the coupler link, the mechanism is a double-rocker Mechanism.

Types of Kinematic Chain:

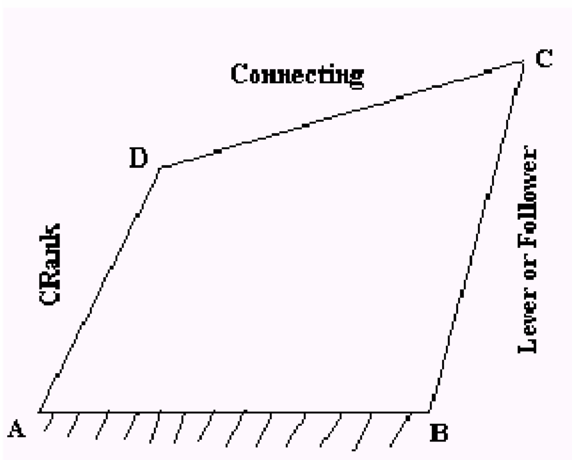
- Four bar chain (or) quadratic cycle chain (all four turning pairs)
- Slider Crank Chain (three turning and are sledding pair)
- Double slider crank chain (Two turning and two sliding pair)

Kinematic Inversions of Mechanisms:

This method of obtaining different mechanisms by fixing different links in a Kinematic chain, it is known as inversion mechanism.

Inversion of Four bar Chain:

A four bar chain consists of 4 turning pairs. It is the basic chain and the diagram is given here for reference.



1) Beam Engine: (Crank and Lever Mechanism)

- A part of the mechanism of a beam engine also known as crank and lever mechanism) which consists of four links is shown in Figure. In this mechanism, when the crank rotates about the fixed center the lever oscillates about a fixed center D.
- The end E of lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

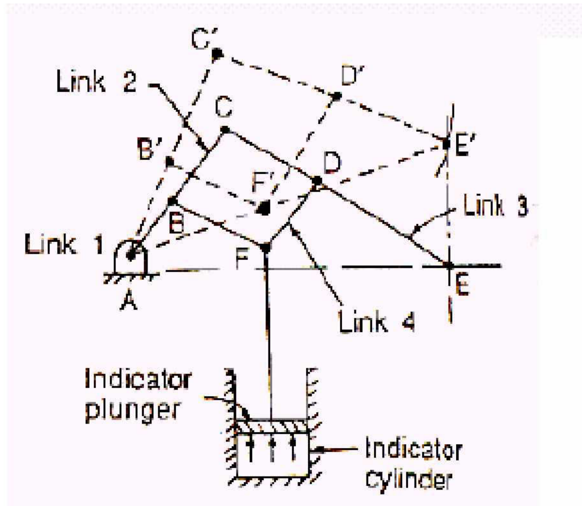
2) Coupling rod of a locomotive (Double Crank mechanism).

- The mechanism of a coupling rod of a locomotive which consists of 4 links is shown in Figure. In this mechanism, the links AD and BC (having equal length) act as crank and are connected to the respective wheels.

- The links CD act as coupling rod and link AB is fixed in order to maintain constant center to center distance between them. This mechanism is meant for transmitting rotary motion from one wheel together wheel.

3. Watt's Indicator Mechanism (Double lever mechanism).

- A watt is indicator mechanism (also known as watt's straight line mechanism or double lever mechanism) which consists of four links.
- The four links are fixed link at A, link AC link CE and link BFD. It may be noted that BF and FD forms one link because these two parts have no relative motion between them.



- The links CE and BFD act as levers. The displacement of link BFD is directly proportional the pressure of gas or stream which acts on the indicator plunger. On any small displacement of mechanism, the tracing point at the end of the link CE traces out approximately a straight line.

Single Slider - Crank - Chain:

- A single crank chain is a modification of the basic four-bar chain. It consists of one sliding pair and three turning pairs. It is found in reciprocating steam engine mechanism.
- This type of mechanism converts rotary motion into reciprocating motion and vice versa.

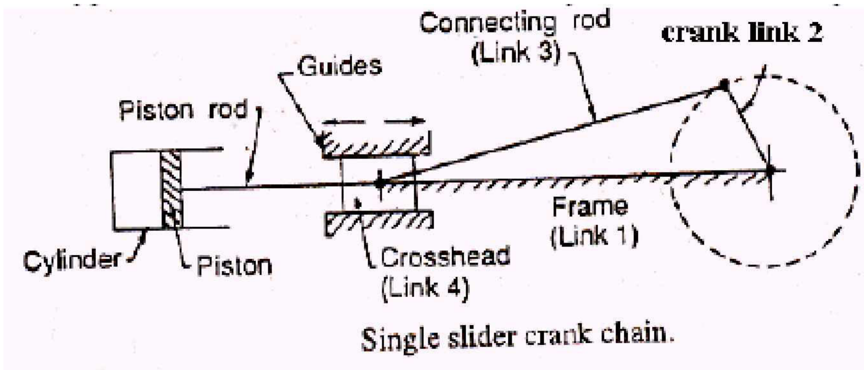
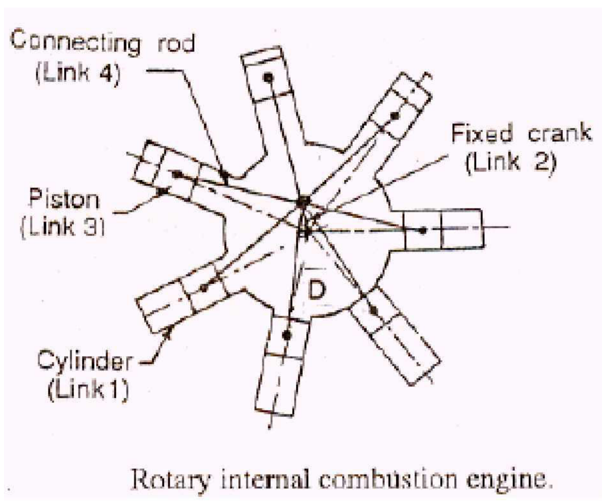


Fig. Inversions of single slider crank chain

3. Rotary Internal Combustion engine or gnome engine:

- Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines used in its place. In consists of seven cylinders in one plane and all revolves about a fixed center. A, as shown in Figure.
- While the crank (link 2) is fixed. In this mechanism when connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinder forming link I.



Double - Slider Crank Chain:

- A Kinematic pair which consists of two turning pairs and two sliding pairs is known as double slider crank chain. Inversions of double slider crank chain:

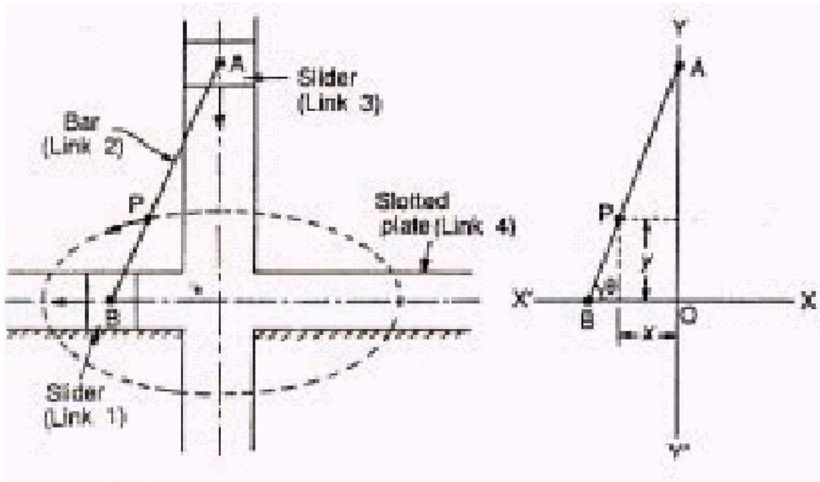
1. Elliptical Trannels:

- This inversion is obtained by fixing the slotted plate (link 4) as in Figure.

$$\cos\theta = \frac{x}{BC}$$

$$\sin\theta = \frac{y}{AC}$$

$$\frac{x^2}{BC^2} + \frac{y^2}{AC^2} = 1$$

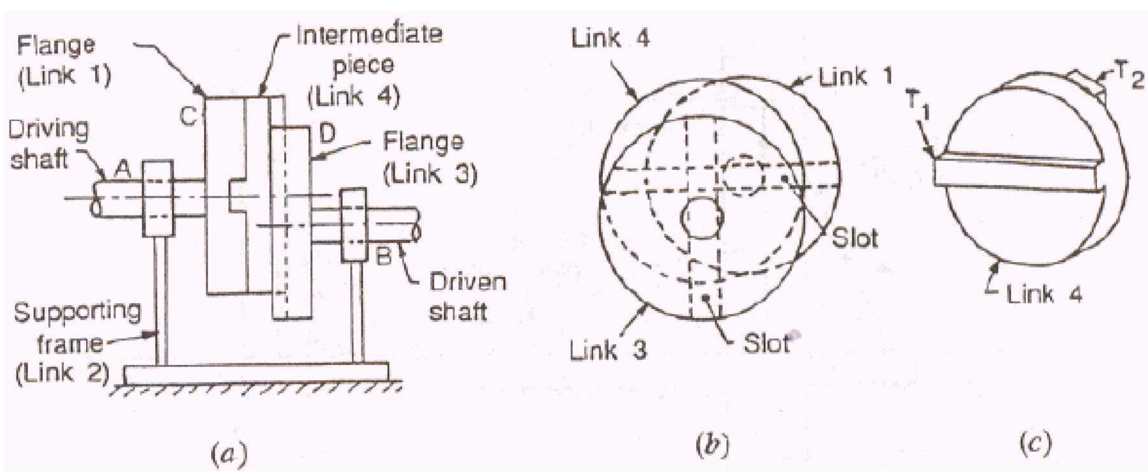


2. Scotch yoke Mechanism:

- This mechanism is used for converting rotary motion in reciprocating motion. The inversion is obtained by fixing either link 1 or link 3.

3. Oldhem's coupling:

- An oldhem's coupling is used for connecting two parallel shafts whose axes are not at a small distance apart. The shafts are coupled in such way that if one shaft rotates, the other shaft also rotates at the same speed.
- This inversion is obtained by fixing links 2 as shown in fig. The shafts to be connected have two flanges (link 1 and link 2) rigidly fastened at their ends by forging.



Oldham's coupling.

Let,

ω- angular velocity each shaft in rad / sec.

R = distance between axis of the shaft in (m)

Mechanical Advantage:

- It is defined as the ratio of the load to effort. in a four bar chain as shown in Figure. The link DA is called the driving link and the link CB as the driven link.
- The force FA acting at A is the effort and the force FB at B will be the load or resistance to over come. We know from the principle of conservation of energy neglecting effect of friction.

$$F_A \times V_A = F_B \times V_B \qquad or \frac{F_B}{F_A} = \frac{V_A}{V_B}$$

ideal mechanical advantage

$$M.A(Ideal)\ \frac{F_B}{F_A} = \frac{V_A}{V_B}$$

v If we consider the effect of friction, less resistance will be over come with the given effort. Therefore actual mechanical advantage will be less.

Actual mechanical advantage:

Mechanism the of Efficiency

$$M.A_{(actual)} = \eta \times \frac{F_B}{F_A} = \eta \times \frac{V_A}{V_B}$$

v Mechanical advantage may also be defined as the ratio of output torque to the input torque.

v Let

T_A = Driving torque

T_B = Resisting torque

ω_A and ω_B = Angular velocity of the driving and driven links respectively.

ideal Mechanical advantage

$$M.A_{(ideal)} = \frac{T_B}{T_A} = \frac{W_A}{W_B}$$

Actual mechanical advantage

$$M.A_{actual)} = \eta \times \frac{T_B}{T_A} = \eta \frac{w_A}{w_B}$$

Transmission Angle:

- In the figure, if *AB* is the input link, the force applied to the output link, *CD*, is transmitted through the coupler link *BC*.
- (That is, pushing on the link *CD* imposes a force on the link *AB*, which is transmitted through the link *BC*.) For sufficiently slow motions (negligible inertia forces), the force in the coupler link is pure tension or compression (negligible bending action) and is directed along *BC*.
- For a given force in the coupler link, the torque transmitted to the output bar (about point *D*) is maximum when the angle between coupler bar *BC* and output bar *CD* is /2. Therefore, angle BCD is called **transmission angle**.

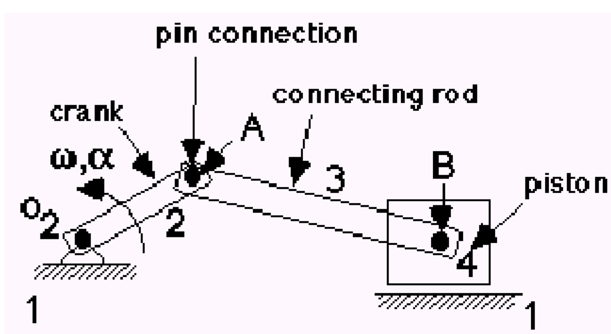
$$\alpha_{max} = |90^\circ - \beta|_{min} < 50^\circ$$

- When the *transmission angle* deviates significantly from /2, the torque on the output bar decreases and may not be sufficient to overcome the friction in the system. For this reason, the **deviation angle** =| /2- | should not be too great.
- In practice, there is no definite upper limit for, because the existence of the inertia forces may eliminate the undesirable force relationship that is present under static conditions. Nevertheless, the following criterion can be followed.

Description of Common Mechanism:

Single slider mechanism:

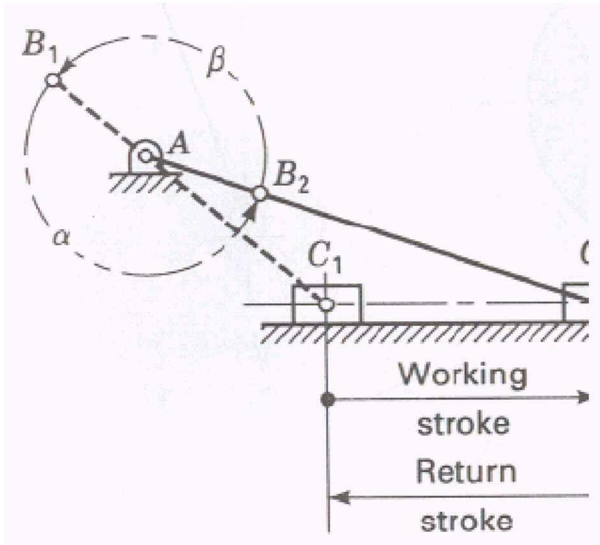
- v A single crank chain is a modification of the basic four-bar chain. It consists of one sliding pair and these turning pair. It is found in reciprocating steam engine mechanism.
- v This type of mechanism converts rotary motion into reciprocating motion and vice versa.



Slider Crank Mechanism

Double slider mechanism:

- A Kinematic pair, which consists of two turning pairs and two sliding pairs, is known as double slider crank chain. Mechanism comprising double slider chain is called double slider mechanism.



(a)

Offset Slider Mechanism:

- The offset slider-crank mechanism shown in Figure has velocity characteristics, which differ from a center slider and crank. If connecting rod of a center slider crank mechanism is large relative to the length of cranks 2 then the resulting motion is very nearly harmonic.

Quick Return Mechanism:

Crank and slotted lever quick return mechanism.

- This mechanism is mostly used in shaping machines, slotting machines, and in rotary internal combustion engines. In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine.
- The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.
- A short link PR transmits the motion from the AP to which carries the tool and reciprocates along the line of stroke R_1 R_2. The line of stroke of the ram (i.e. R_1 R_2) is perpendicular to AC produced.

- In the extreme positions, AP_1, and AP_2 are targeted to the circle and the tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from position CB_1 CB_2 (or through an angle B) in clockwise when the crank rotates from the position CB_2 to CB_1 (or through angle) in the clockwise direction. Since the crank has uniform angles speed, therefore.

$$\frac{Time\ of\ Cutting\ stroke}{Time\ of\ return\ stroke} = \frac{\beta}{\alpha} = \frac{\beta}{360_{\beta}}$$

$$or\ \frac{360^{\circ} - \alpha}{\alpha}$$

- Since the tool travels a distance of R_1 R_2 during cutting and return stroke, therefore travel of the tool or length stroke.

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 Sin\angle P_1AQ$$

$$= 2AP_1 Sin\left(90 - \frac{\alpha}{2}\right) = 2AP\ Cos\frac{\alpha}{2}$$

$$(\therefore AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \qquad (\therefore COS\frac{\alpha}{2} = \frac{CB_1}{AC}$$

$$2AP \times \frac{CB}{AC} \qquad (\therefore CB_1 = CB)$$

Whitworth quick return motion mechanism:

- This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed.

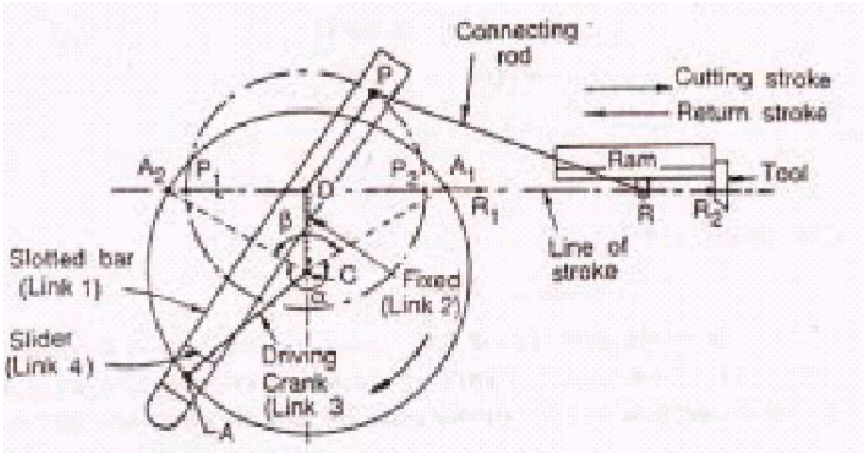


Fig- whit worth quick return motion mechanism

- The similar end attached to the crank pin at A slides along the slotted bar PA (link1) which oscillates at a pointed point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced i.e. along a line passing through D and perpendicular to CD. When the driving crank

- CA moves from the position CA_1 to CA_2 (or the link DP from the portion DP_1 toDP_2) through an angle . in the clock wise direction, the tool moves from the left hand end to its stroke to the right hand end through a distance 2PD. Now when the driving crank moves from the position CP_2 to CP_1 (or the link DP from DP_1 to DP_2) through an angle P in the clockwise direction, the tool moves back from the right hand end of its stroke to the left hand end.

- A little consideration will show that the line taken during the left to right movement of the ram (i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1, CA_2.
- Similarly, time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2.
- Since the crank link CA rotates a uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke.

- In other words the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return stroke is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \; or \; \frac{360^\circ - \beta}{\beta}$$

In order to find the length of the effective stroke.

R_1R_2, mark $P_1R_1=P_2R_2$ = PR. The length of effective stroke is also equal to 2PD.

Snap - Action Mechanisms:

- The mechanism shown in figure is typical of snap action mechanisms. They also include spring clips and circuit breakers. Typical snap-action, toggle, or flip-flop mechanisms used for switches, clamps, or fasteners.
- One elements of a mechanism are always numbered beginning with 1 for the base or frame, and 2 for the input or driving element. The mechanism of part *(a)* is bitable; that of *(b)* is a true toggle.

Linear Actuators:

Linear actuators include

- Stationery screw with traveling nuts
- Stationery nuts with traveling screws
- Single all double acting hydraulic and pneumatic cylinders.

Motion Adjustment and Damping mechanisms:

Fine Adjustments.

§ Fine adjustments may be obtained with screws, including the differential screws, worm gearing, wedges, lever and various motions adjusting mechanism.

Clamping Mechanism:

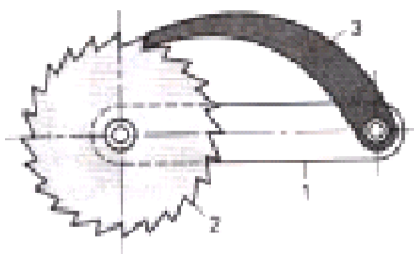
§ Typical clamping mechanism are the C-clamp, the woodworker's screw clamp, cam and lever actuated clamps, vises, presses such as the toggle press, collets and stamp mills.

Location Mechanism

§ These are usually self-centering and locate either axially or angularly using springs or détentes.

Ratchets and escapements.

- These are May different forms of ratchets and escapements, some quite clever. They are used in lock, jacks, clockwork and other application requiring same form of intermittent motion figure illustrates four typical applications.



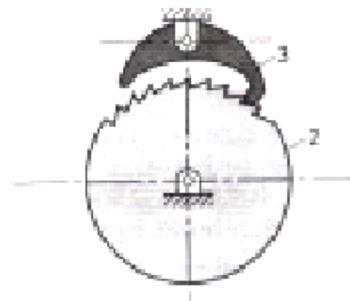
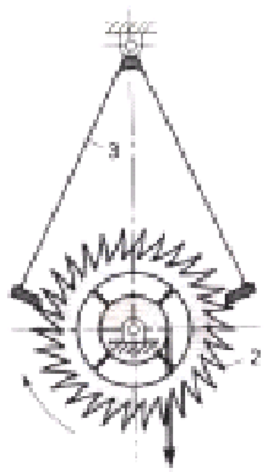


Fig (a): one direction of rotation of wheel 2 Fig **(b)**: Escapements used Rotary adjustments.



Fig(c)

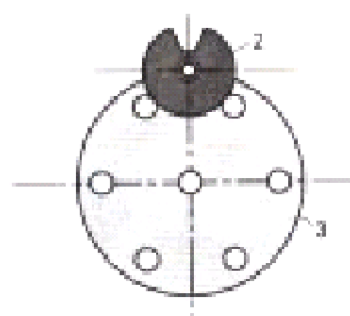


fig (d)

Fig(c): To regulate movement of clockwork.

Fig (d): Control wheel 2 which may rotate continuously to allow wheel 3 to be driven.

Indexing Mechanism:

- The indexes of Figure (a) uses standard gear teeth; for light loads, pins can be used in wheel 2 with corresponding slots in wheel 3, but neither form should be used if the shaft inertias large.

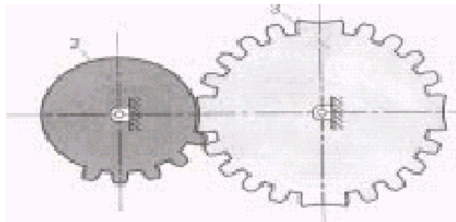
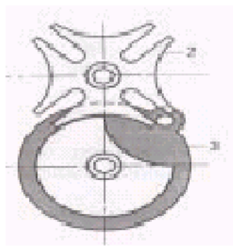
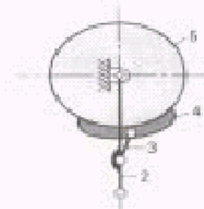


Fig (a)

- Figure (b) indicates a Geneva wheel indexer. Three or more slots (up to 16) may be used in driver 2 and wheel 3 can be geared to the output to be indexed. High speeds and large inertias may cause problems with this indexer.
- Toothless ratchet 5 in Fig C is driven by the oscillating crank 2 of variable throw.



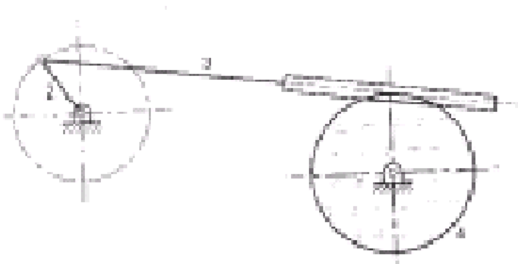
(b)



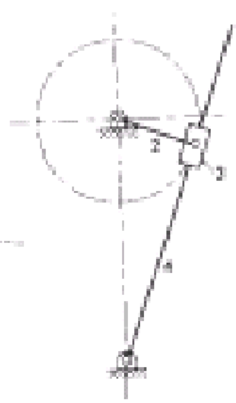
(c)

Swinging or Rocking Mechanism:

- The class of swinging or rocking mechanisms is often termed as oscillators; in each case the output member rocks or swings through engines, which are generally less than 360 . However output shaft can be geared to a second shaft to produce larger angles of oscillation.



(a)



(b)

Figure (a) is a mechanism consisting of rotating crank 2 and a couple 3 containing rock which meshes with output gear 4 to produce the oscillating motion.

In Figure (b) crank 2 drives member 3, which slides on output link 4, producing a rocking motion. This mechanism is quick - return linkage because crack 2 rotates through a large angle on the forward stroke of link 4 than on the return stroke.

(b) (d)

Figure C is a four bar linkage called the crank and rocker mechanism cranks 2 drives rocker 4 through coupler 3 of course, link 1 is the frame. The characteristics of the rocking motion depend on the dimensions of the links and the placement of the frame points.

Figure D. Illustrates cam and follower mechanism, in which the rotating 2 drives, link 3 called the follower in a rocking motion.

Straight Line Generations:

- In the late seventeenth century before the development of the milling machine it was extremely difficult to machine straight flat surfaces. For this reason good prismatic pairs without backlash were not easy to make.
- During that era much thought was given to the problem of attaining a straight line motion as a part of the coupler curve of a having only revolute connections.
- Probably the best known result of this search is the straight line mechanism development by Watt for guiding the pistons of early steam engines.

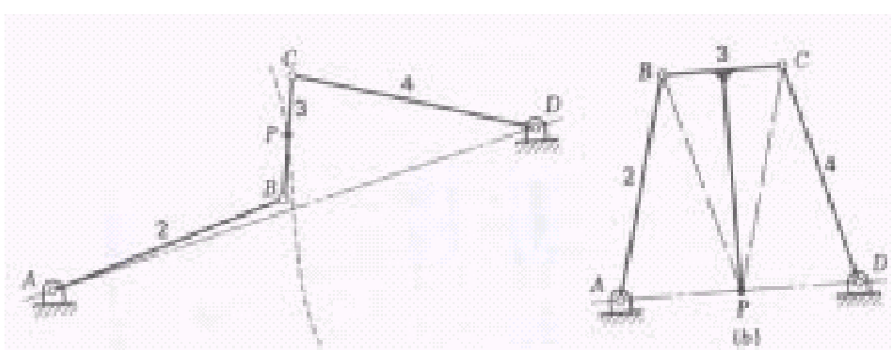


Fig (a) fig (b)

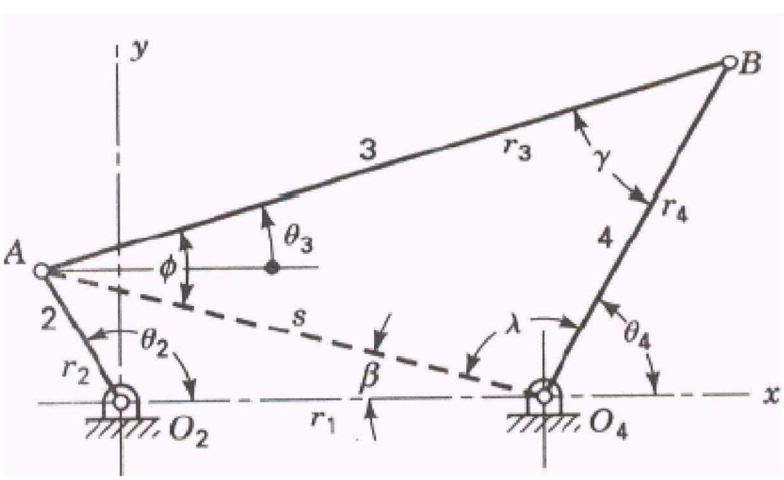
Figure (a) show Watt's linkage to four bar linkage developing an approximate straight line as a part of its coupler curve. Although it does not generate an exact straight line, a good approximation is achieved over a considerable distance of travel. Other such linkages shown in figure are

- Robert's mechanisms (Fig b)
- Chebychev linkage (Fig c)
- Peaucillier inversor (Fig d)

It also includes, Robots speed changing devices, and computing mechanism function generates loading mechanism and transportation devices under this classification.

Design of Crank -Rocker Mechanism:

- v The four -bar linkage shown in figure is called the crank- rocker mechanism, link 2 is called the crank can rotate in a full circle but the rocker, link 4 can only oscillate.
- v We shall generally follow the accepted practice of designating the frame or fixed link as link 1. Line 3 in figure is called the coupler or connecting rod.
- v With the four bar linkage the position problem generally consists of finding the positions of the coupler and output links or rocker when the dimensions of all the members are given together with the crank position, to obtain the analytical solution we designate S as the distance AO4 in Figure.
- v The cosine law can then be written twice for each of the two triangles O4O2A and ABO4. In terms of the angles and link lengths shown in the figure, we then have:



$$S = \left[r_1^{\ 2} + r_2^{\ 2} - 2r_1r_2 \ Cos\theta_2\right]^{\frac{1}{2}}$$

$$\beta = Cos^1 \frac{r_1^{\ 2} + S^2 - r_2^{\ 2}}{2r_1S}$$

$$\psi = Cos^{-1} \frac{r_3^{\ 2} + S^2 - r_4^{\ 2}}{2r_3S}$$

$$\lambda = Cos^{-1} \frac{r_4^{\ 2} + S^2 - r_3^{\ 2}}{2r_4S}$$

$$\theta_3 = \psi - \beta$$

$$\theta_4 = \Pi\text{-}\lambda\text{-}\beta$$

$$\theta_3 = \psi + \beta$$

$$\theta_4 = \pi - \lambda + \beta$$

- Finally it is worth noting that the Equations 3 and 4 yield double valves too since they are arc cosines.
- These will always be positive and negative pairs of values, the positive values to correspond the open configuration shown, while the negatives values correspond to the crossed closure.

UNIT II

KINEMATICS - VELOCITY AND ACCELERATION DIAGRAMS

SYLLUBUS:

"Displacement, velocity and acceleration - analysis in simple mechanisms - Graphical Method velocity and acceleration polygons - Kinematic analysis by Complex Algebra methods-Vector Approach, Computer applications in the Kinematic analysis of simple mechanisms-Coincident points-Coriolis Acceleration".

<u>Content:</u>

- Describe a mechanism.
- Define relative and absolute velocity.
- Define relative and absolute acceleration.
- Define radial and tangential velocity.
- Define radial and tangential acceleration.
- Describe a four bar chain.
- Solve the velocity and acceleration of points within a mechanism.
- Use mathematical and graphical methods.
- Construct velocity and acceleration diagrams.
- Define the Coriolis Acceleration.
- Solve problems involving sliding links.

It is assumed that the student is already familiar with the following concepts.

- Vector diagrams.
- Simple harmonic motion.
- Angular and linear motion.
- Inertia force.
- Appropriate level of mathematics.

All these above may be found in the pre-requisite tutorials.

INTRODUCTION

A mechanism is used to produce mechanical transformations in a machine. This transformation could be any of the following.

- It may convert one speed to another speed.
- It may convert one force to another force.
- It may convert one torque to another torque.
- It may convert force into torque.
- It may convert one angular motion to another angular motion.
- It may convert angular motion into linear motion.
- It may convert linear motion into angular motion.

A good example is a crank, connecting rod and piston mechanism.

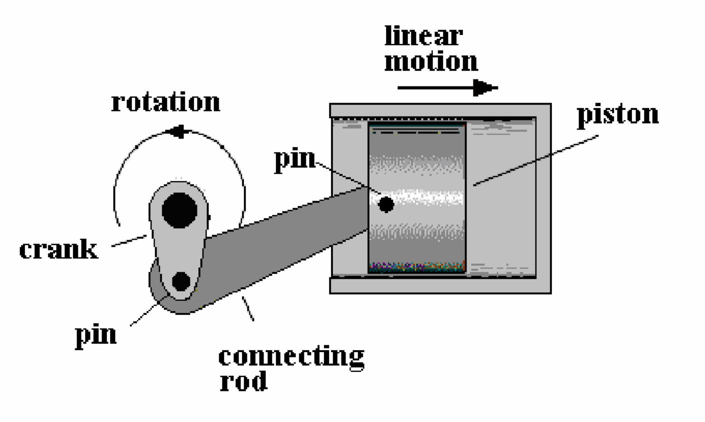


Figure 1

- If the crank is turned, angular motion is converted into linear motion of the piston and input torque is transformed into force on the piston.
- If the piston is forced to move, the linear motion is converted into rotary motion and the force into torque. The piston is a sliding joint and this is called ***PRISMATIC*** in some fields of engineering such as robotics.
- The pin joints allow rotation of one part relative to another. These are also called ***REVOLUTE*** joints in other areas of engineering.

Consider the next mechanism used in shaping machines and also known as the Whitworth quick- return mechanism.

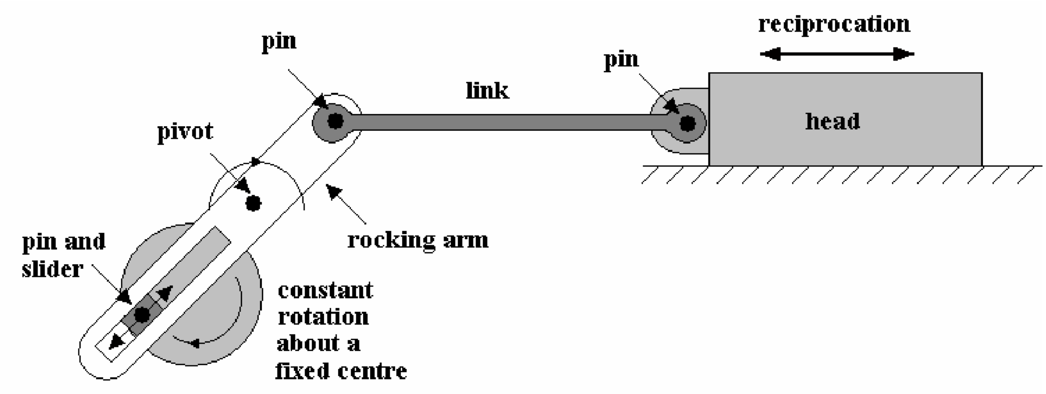


Figure 2

- The input is connected to a motor turning at constant speed. This makes the rocking arm move back and forth and the head (that carries the cutting tool) reciprocates back and forth.
- Depending on the lengths of the various parts, the motion of the head can be made to move forwards at a fairly constant cutting speed but the return stroke is quick.
- Note that the pin and slider must be able to slide in the slot or the mechanism would jam. This causes problems in the solution because of the sliding link and this is covered later under Coriolis acceleration.
- The main point is that the motion produced is anything but simple harmonic motion and at any time the various parts of the mechanism have a displacement, velocity and acceleration.
- The acceleration gives rise to inertia forces and this puts stress on the parts in addition to the stress produced by the transmission of power.
- For example the acceleration of a piston in an internal combustion engine can be enormous and the connecting rod is subjected to high stresses as a result of the inertia as well as due to the power transmission.
- You will find in these studies that the various parts are referred to as links and it can be shown that all mechanisms are made up of a series of four links.
- The basic four bar link is shown below. When the input link rotates the output link may for example swing back and forth. Note that the fourth link is the frame of the machine and it is rigid and unable to move.
- With experience you should be able to identify the four bar chains in a mechanism. All the links shown are rigid links which means they may push or pull. It is possible to have links made of chain or rope which can only pull.

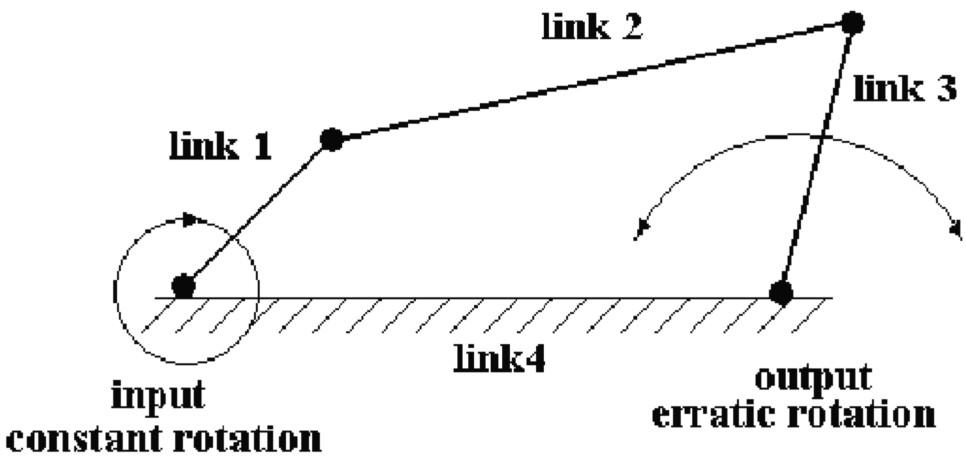


Figure 3

2. DISPLACEMENT, VELOCITY AND ACCELERATION

- All parts of a mechanism have displacement, velocity and acceleration. In the tutorial on free vibration, a mechanism called the Scotch Yoke was examined in order to explain sinusoidal or harmonic motion.
- The wheel turns at a constant speed and the yoke moves up and down.

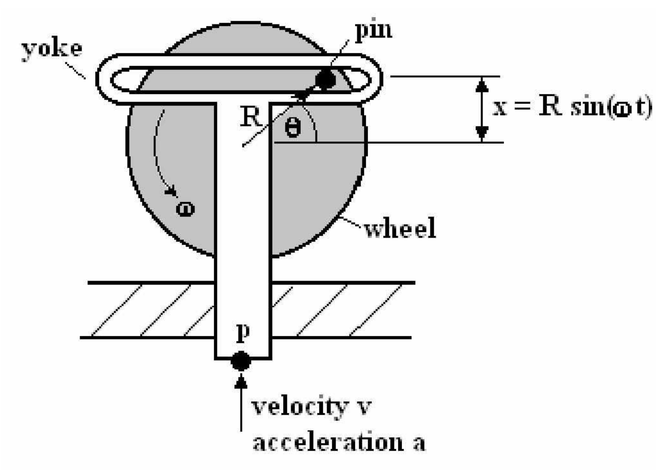


Figure 4

It was shown that the displacement 'x', velocity 'v' and acceleration 'a' of point p was given as follows. Angle $\theta = \omega t$

$$\text{Displacement } x = R \sin(\omega t).$$

$$\text{Velocity } v = dx/dt = \omega R \cos(\omega t)$$

$$\text{Acceleration } a = dv/dt = -\omega^2 R \sin(\omega t)$$

- The values can be calculated for any angle or moment of time. The acceleration could then be used to calculate the inertia force needed to accelerate and decelerate the link.
- Clearly it is the maximum values that are needed. Other mechanisms can be analyzed mathematically in the same way but it is more difficult.
- The starting point is to derive the equation for displacement with respect to angle or time and then differentiate twice to get the acceleration.
- Without the aid of a computer to do this, the mathematics is normally much too difficult and a graphical method should be used as shown later.

3. VELOCITY DIAGRAMS

This section involves the construction of diagrams which needs to be done accurately and to a suitable scale. Students should use a drawing board, ruler, compass, protractor and triangles and possess the necessary drawing skills.

ABSOLUTE AND RELATIVE VELOCITY

An absolute velocity is the velocity of a point measured from a fixed point (normally the ground or anything rigidly attached to the ground and not moving). Relative velocity is the velocity of a point measured relative to another that may itself be moving.

TANGENTIAL VELOCITY

Consider a link A B pinned at A and revolving about A at angular velocity ω. Point B moves in a circle relative to point A but its velocity is always tangential and hence at 90^o to the link. A convenient method of denoting this tangential velocity is $(v_B)_A$ meaning the velocity of B relative to A. This method is not always suitable.

Figure 5

RADIAL VELOCITY

- Consider a sliding link C that can slide on link AB. The direction can only be radial relative to point A as shown.
- If the link AB rotates about A at the same time then link C will have radial and tangential velocities.

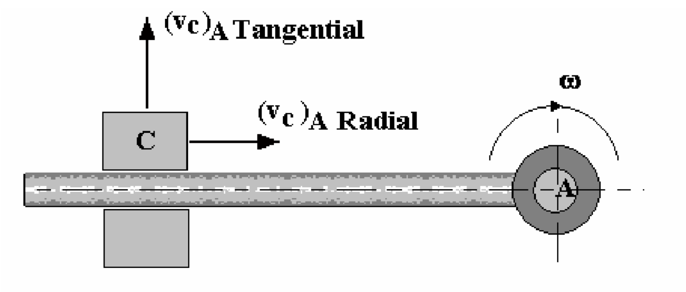


Figure 6

- Note that both the tangential and radial velocities are denoted the same so the tags radial and tangential are added.
- The sliding link has two relative velocities, the radial and the tangential. They are normal to each other and the true velocity relative to A is the vector sum of both added as shown.
- ***Note that lower case letters are used on the vector diagrams.*** The two vectors are denoted by c_1 and c_2. The velocity of link C relative to point A is the vector a c_2.

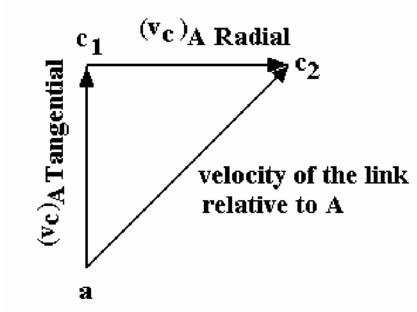


Figure 7

CRANK, CONNECTING ROD AND PISTON

Consider this mechanism again. Let's freeze the motion (snap shot) at the position shown. The diagram is called a space diagram.

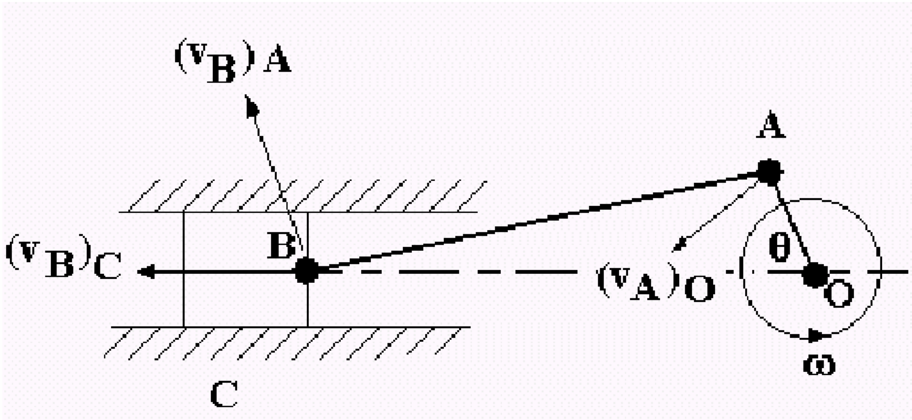


Figure 8

- Every point on every link has a velocity through space. First we label the centre of rotation, often this is the letter O. Point A can only move in a tangential direction so the velocity of A relative to O is also its absolute velocity and the vector is normal to the crank and it is designated $(v_A)_O$. (Note the rotation is anticlockwise).
- Now suppose that you are sat at point A and everything else moves relative to you. Looking towards B, it would appear the B is rotating relative to you (in reality it is you that is rotating) so it has a tangential velocity denoted $(V_B)_A$.
- The direction is not always obvious except that it is normal to the link.Consider the fixed link OC. Since both points are fixed there is no velocity between them so $(v_c)_O = 0$.
- Next consider that you at point C looking at point B. Point B is a sliding link and will move in a straight line in the direction fixed by the slider guides and this is velocity $(v_B)_C$. It follows that the velocity of B seen from O is the same as that seen from C so $(v_B)_C = (v_B)_O$.
- The absolute velocity of B is $(v_B)_C = (v_B)_O$ and this must be the vector sum of $(V_A)_O$ and $(v_B)_A$ and the three vectors must form a closed triangle as shown. The velocity of the piston must be in the direction in which it slides (conveniently horizontal here). This is a velocity diagram.

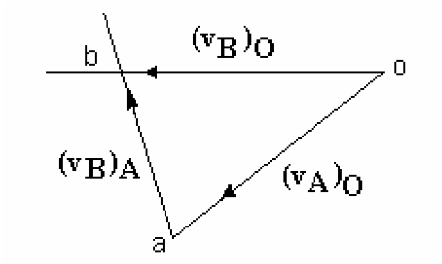


Figure 9

- First calculate the tangential velocity $(v_A)_O$ from v = ω x radius = ω x OA
- Draw the vector o - a in the correct direction (note lower case letters).
- We know that the velocity of B relative to A is to be added so the next vector ab starts at point a. At point a draw a line in the direction normal to the connecting rod but of unknown length.
- We know that the velocity of B relative and absolute to O is horizontal so the vector ob must start at a. Draw a horizontal line (in this case) through o to intersect with the other line. This is point b. The vectors ab and ob may be measured or calculated. Usually it is the velocity of the slider that is required.
- In a design problem, this velocity would be evaluated for many different positions of the crank shaft and the velocity of the piston determined for each position.
- Remember that the slider direction is not always horizontal and the direction of o - b must be the direction of sliding.

<u>WORKED EXAMPLE No.1</u>

The mechanism shown has a crank 50 mm radius which rotates at 2000 rev/min. Determine the velocity of the piston for the position shown. Also determine the angular velocity of link AB about A.

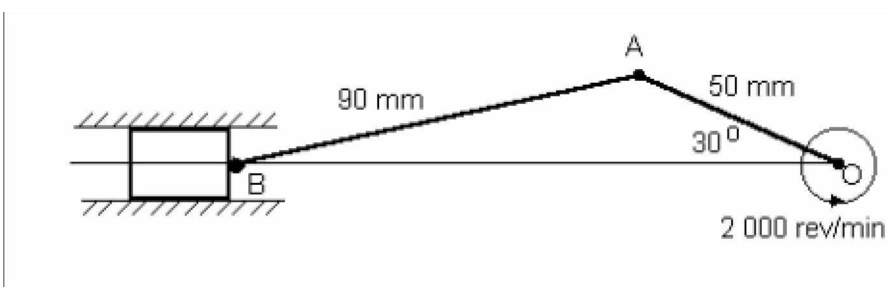


Figure 10

SOLUTION

- Note the diagrams are not drawn to scale. The student should do this using a suitable scale for example 1 cm = 1 m/s.
- This is important so that the direction at 90° to the link AB can be transferred to the velocity diagram.
- Angular speed of the crank $\omega = 2\pi N/60 = 2\pi \times 2000/60 = 209.4$ rad/s

$$(v_A)_O = \omega \times \text{radius} = 209.4 \times 0.05 = 10.47 \text{ m/s.}$$

First draw vector oa. (Diagram a)

- Next add a line in the direction ab (diagram b)
- Finally add the line in the direction of ob to find point b and measure ob to get the velocity.

(Diagram C).

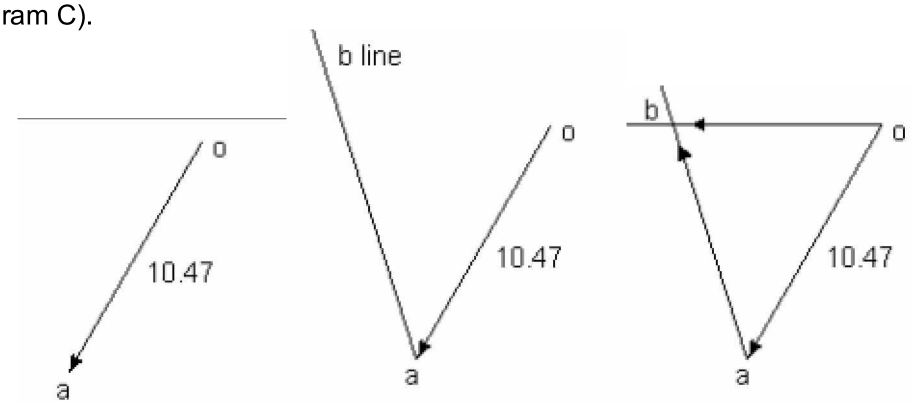


Figure 11a Figure 11b Figure 11c

- The velocity of B relative to O is 7 m/s.
- The tangential velocity of B relative to A is the vector ab and this gives 9.2 m/s.
- The angular velocity of B about A is found by dividing by the radius (length of AB).
- ω for AB is then 9.2/0.09 = 102.2 rad/s. (note this is relative to A and not an absolute angular velocity)

4.BAR CHAIN

- The input link rotates at a constant angular velocity ω_1. The relative velocity of each point relative to the other end of the link is shown.
- Each velocity vector is at right angles to the link. The output angular velocity is ω_2 and this will not be constant. The points A and D are fixed so they will appear as the same point on the velocity diagram.
- The methodology is the same as before and best shown with another example.

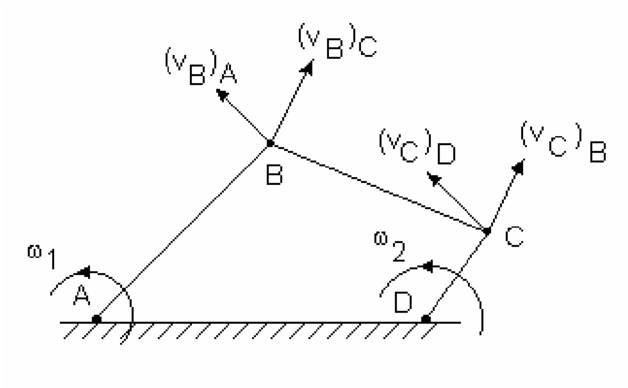


Figure 12

WORKED EXAMPLE No. 2

Find the angular velocity of the output link when the input rotates at a constant speed of 500 rev/min. The diagram is not to scale.

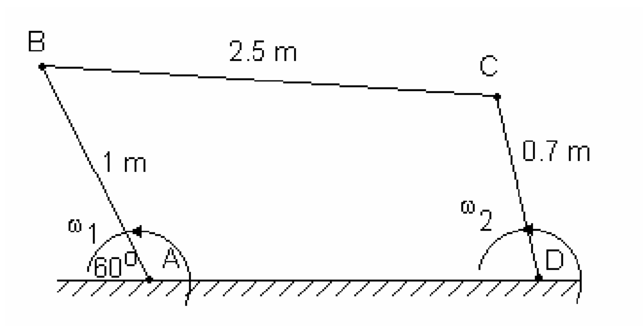


Figure 13

SOLUTION

First calculate ω_1.

$\omega_1 = 2\pi \times 500/60 = 52.36$ rad/s.

Next calculate the velocity of point B relative to A.

$(V_B)_A = \omega_1 \times AB = 52.36 \times 1 = 52.36$ m/s.

Draw this as a vector to an appropriate scale.

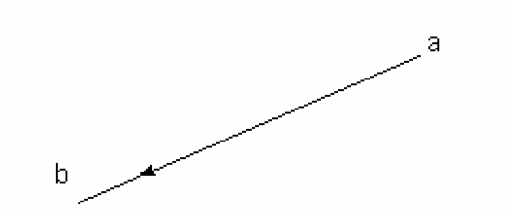


Figure 14a

Next draw the direction of velocity C relative to B at right angles to the link BC passing through point b on the velocity diagram.

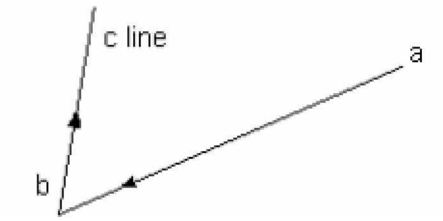


Figure 14 b

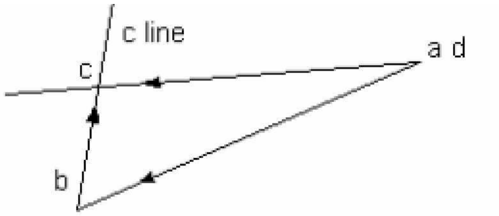


Figure 14 c

- Next draw the direction of the velocity of C relative to D at right angles to link DC passing through point a (which is the same as point d). Point c is where the two lines intersect,
- Determine velocity cd by measurement or any other method. The velocity of point C relative to D and is 43.5 m/s.
- Convert this into angular velocity by dividing the length of the link DC into it.

$$\omega_2 = 43.5/0.7 = 62 \text{ rad/s.}$$

4. ACCELERATION DIAGRAMS

- It is important to determine the acceleration of links because acceleration produces inertia forces in the link which stress the component parts of the mechanism.
- Accelerations may be relative or absolute in the same way as described for velocity.
- We shall consider two forms of acceleration, tangential and radial. Centripetal acceleration is an example of radial.

CENTRIPETAL ACCELERATION

- A point rotating about a centre at radius R has a tangential velocity v and angular velocity ω and it is continually accelerating towards the centre even though it never moves any closer. This is centripetal acceleration and it is caused by the constant change in direction.
- It follows that the end of any rotating link will have a centripetal acceleration towards the opposite end.

The relevant equations are: $v = \omega R$ $a = \omega^2 R$ or $a = v^2/R$.

- The construction of the vector for radial acceleration causes confusion so the rules must be strictly followed. Consider the link AB. The velocity of B relative to A is tangential $(v_B)_A$.
- The centripetal acceleration of B relative to A is in a radial direction so a suitable notation might be a_R. It is calculated using $a_R = \omega \times AB$ or $a_R = v^2/AB$.

Note the direction is towards the centre of rotation but the vector starts at a and ends at b_1. It is very important to get this the right way round otherwise the complete diagram will be wrong.

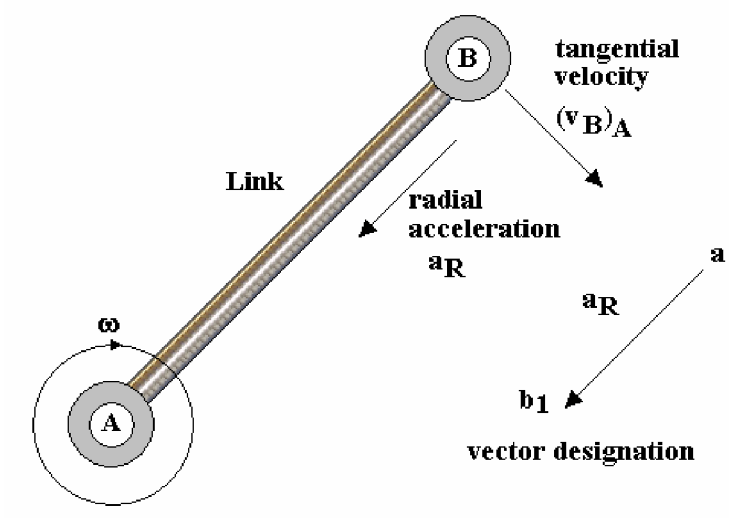


Figure 15

TANGENTIAL ACCELERATION

Tangential acceleration only occurs if the link has an angular acceleration α rad/s^2. Consider a link AB with an angular acceleration about A.

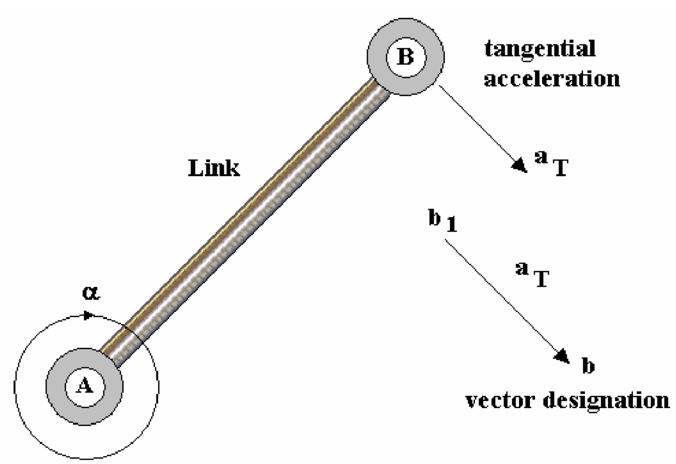


Figure 16

- Point B will have both radial and tangential acceleration relative to point A. The true acceleration of point B relative to A is the vector sum of them. This will require an extra point. We will use b_1 and b on the vector diagram as shown.
- Point B is accelerating around a circular path and its direction is tangential (at right angles to the link). It is designated a_T and calculated using $a_T = \alpha \times AB$.
- The vector starts at b_1 and ends at b. The choice of letters and notation are arbitrary but must be logical to aid and relate to the construction of the diagram.

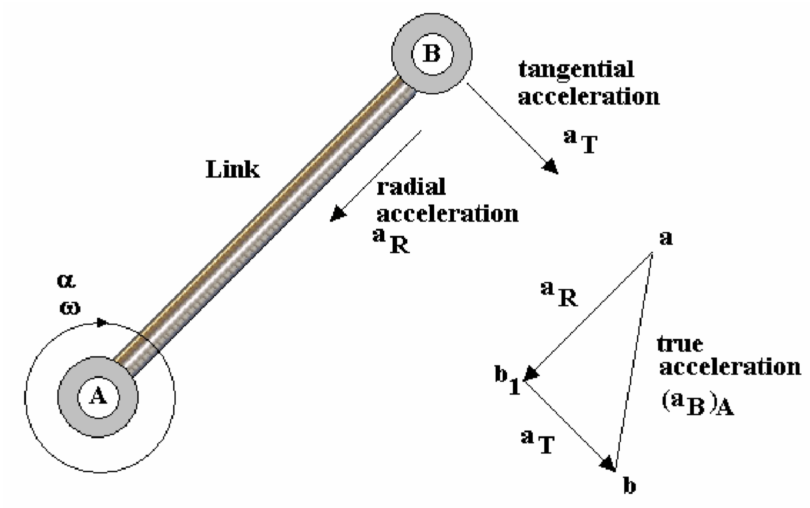


Figure 17

WORKED EXAMPLE No.3

A piston, connecting rod and crank mechanism is shown in the diagram. The crank rotates at a constant velocity of 300 rad/s. Find the acceleration of the piston and the angular acceleration of the link BC. The diagram is not drawn to scale.

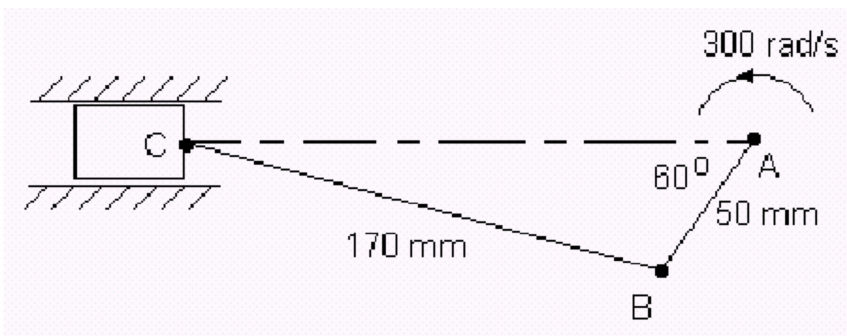


Figure 18

SOLUTION:

First calculate the tangential velocity of B relative to A.

$(v_B)_A$ = ω x radius = 300 x 0.05 = 15 m/s.

Next draw the velocity diagram and determine the velocity of C relative to B.

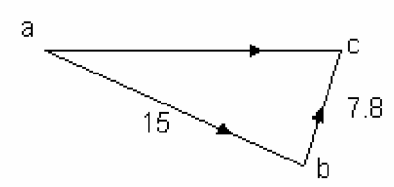


Figure 19

From the velocity diagram $(v_C)_B$ = 7.8 m/s

- Next calculate all accelerations possible and construct the acceleration diagram to find the acceleration of the piston.
- The tangential acceleration of B relative to A is zero in this case since the link has no angular acceleration ($\alpha = 0$).
- The centripetal acceleration of B relative to A

$$a_R = \omega^2 \text{x AB} = 300^2 \text{ x } 0.05 = 4500 \text{ m/s}^2.$$

The tangential acceleration of C relative to B is unknown.

The centripetal acceleration of C to B

$$a_R = v^2/BC = 7.8^2 /0.17 = 357.9 \text{ m/s}^2.$$

The stage by stage construction of the acceleration diagram is as follows.

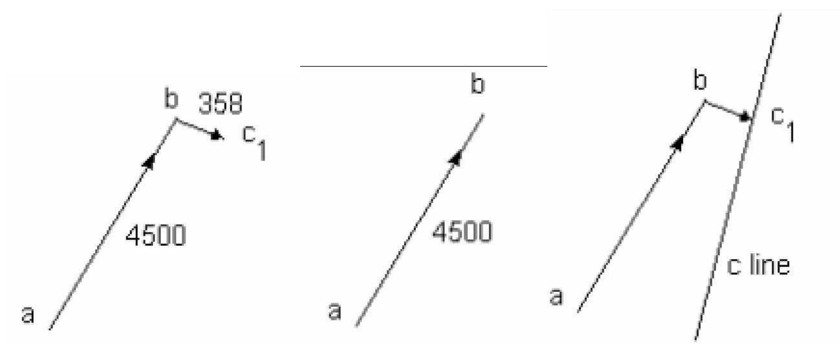


Figure 20a ***Figure 20b*** ***Figure 20c***

- First draw the centripetal acceleration of link AB (Fig.a). There is no tangential acceleration so designate it ab. Note the direction is the same as the direction of the link towards the centre of rotation but is starts at a and ends at b.
- Next add the centripetal acceleration of link BC (Figure b). Since there are two accelerations for point C designate the point c_1. Note the direction is the same as the direction of the link towards the centre of rotation.
- Next add the tangential acceleration of point C relative to B (Figure c). Designate it c_1 c. Note the direction is at right angles to the previous vector and the length is unknown. Call the line a c line.
- Next draw the acceleration of the piston (figure d) which is constrained to be in the horizontal direction. This vector starts at a and must intersect the c line. Designate this point c.

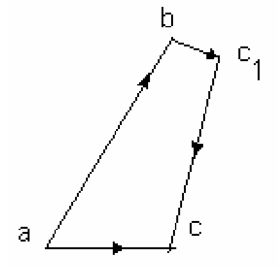


Figure 20d

- The acceleration of the piston is vector ac so $(a_C)_B = 1505\ m/s^2$. The tangential acceleration of C relative to B is $c_1\,c = 4000\ m/s^2$.
- At the position shown the connecting rod has an angular velocity and acceleration about its end even though the crank moves at constant speed.
- The angular acceleration of BC is the tangential acceleration divided by the length BC.

$$\alpha_{(BC)} = 4000 / 0.17 = 23529\ rad/s^2.$$

WORKED EXAMPLE No.4

The diagrams shows a "rocking lever" mechanism in which steady rotation of the wheel produces an oscillating motion of the lever OA. Both the wheel and the lever are mounted in fixed centers. The wheel rotates clockwise at a uniform angular velocity (ω) of 100 rad/s. For the configuration shown, determine the following.

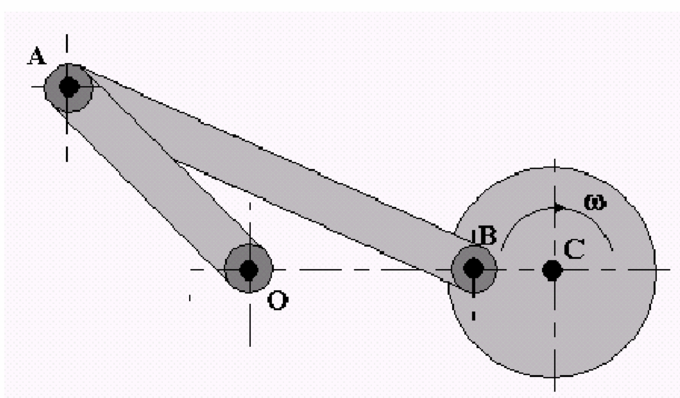


Figure 21

(i) The angular velocity of the link AB and the absolute velocity of point A.

(ii) The centrifugal accelerations of BC, AB and OA.

(iii)The magnitude and direction of the acceleration of point A.

The lengths of the links are as follows.

BC = 25 mm AB = 100 mm OA = 50 mm OC = 90 mm

SOLUTION

The solution is best done graphically. First draw a line diagram of the mechanism to scale. It should look like this.

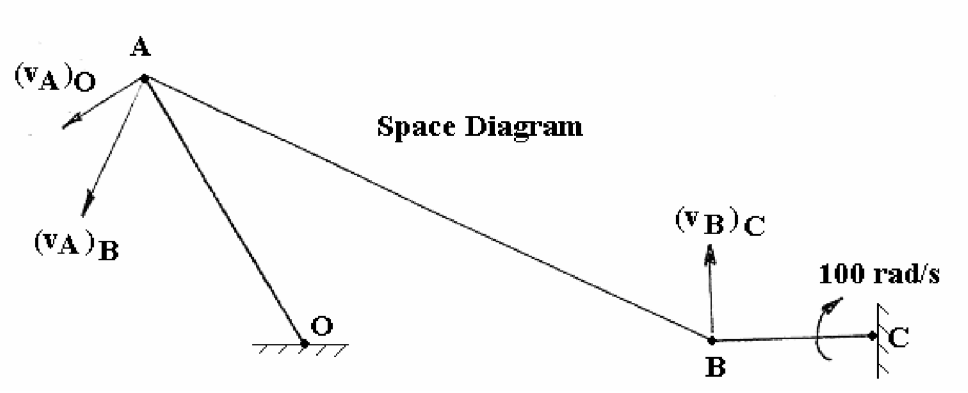


Figure 22

Next calculate the velocity of point B relative to C and construct the velocity diagram.

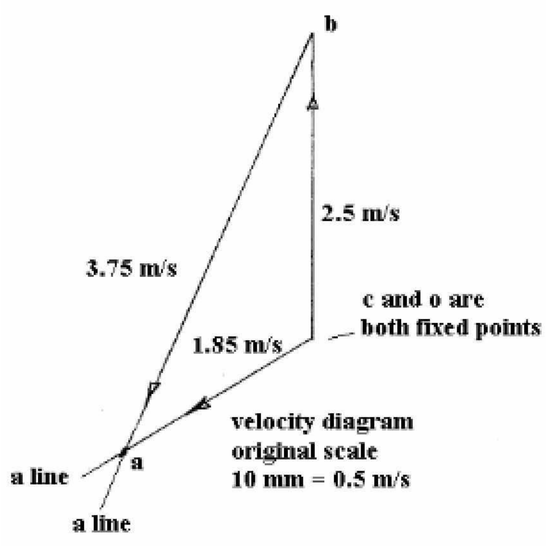


Figure 23

$(v_B)C = \omega \text{ x radius} = 100 \text{ x } 0.025 = 2.5$ m/s

Scale the following velocities from the diagram.

$(v_A)O = 1.85$ m/s {answer (i)} (vA)B = 3.75 m/s

Angular velocity = tangential velocity/radius

For link AB, ω = 3.75/0.1 = 37.5 rad/s. {answer (i)} Next calculate all the accelerations possible.

- Radial acceleration of BC = ω^2 x BC = 100^2 x 0.025 = 250 m/s^2.
- Radial acceleration of AB = v^2/AB = 3.75^2/0.1 = 140.6 m/ s^2.
- Check same answer from ω^2 x AB = 37.5^2 x 0.1 = 140.6 m/ s^2.
- Radial Acceleration of OA is v^2/OA = 1.85^2/0.05 = 68.45 m/ s^2.

Construction of the acceleration diagram gives the result shown.

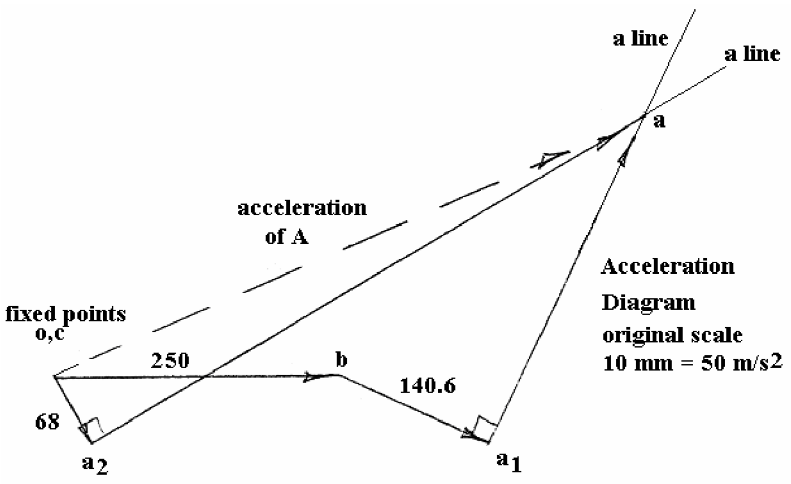


Figure 24

The acceleration of point A is the vector o- a shown as a dotted line. Scaling this we get 560 m/s^2.

<u>WORKED EXAMPLE No. 5</u>

Find the angular acceleration of the link CD for the case shown.

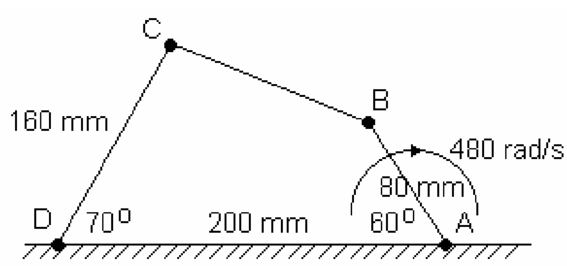


Figure 25

SOLUTION

First calculate or scale the length CB and find it to be 136 mm.

Next find the velocities and construct the velocity diagram. Start with link AB as this has a known constant angular velocity.

$$(v_B)_A = \omega \text{ x radius} = 480 \text{ x } 0.08 = 38.4 \text{ m/s}$$

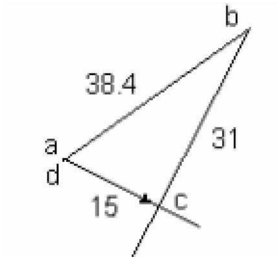
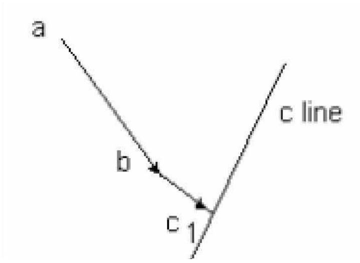
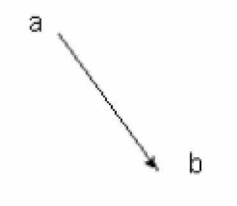


Figure 26

1. Next calculate all the accelerations possible.
2. The centripetal acceleration of B to A is $38.4^2/0.08$ = 18 432 m/s^2
3. The centripetal acceleration of C to D is $15^2/0.16$ = 1406 m/s^2
4. The centripetal acceleration of C to B is $31^2/0.136$ = 7066 m/s^2.
5. We cannot calculate any tangential acceleration at this stage.
6. The stage by stage construction of the acceleration diagram follows.
7. First draw the centripetal acceleration of B to A (Figure a). There is no tangential to add on).





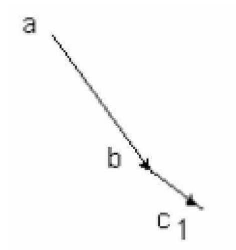
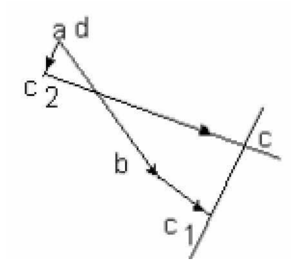


Figure 27a ***Figure 27b*** ***Figure 27c***

8. Next add the centripetal acceleration of C to B (figure b)
9. Next draw the direction of the tangential acceleration of C to B of unknown length at right angles to the previous vector (figure c). Designate it as a c line.
10. We cannot proceed from this point unless we realize that points a and d are the same (there is no velocity or acceleration of D relative to A). Add the centripetal acceleration of C to D (figure d). This is 1406 m/s^2 in the direction of

link CD. Designate it d c_2.



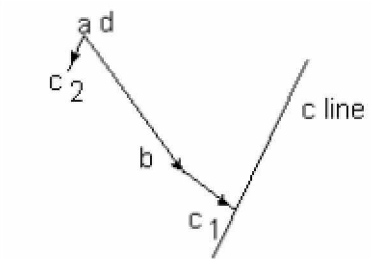


Figure 27d ***Figure 27e***

Finally draw the tangential acceleration of C to D at right angles to the previous vector to intersect the c line (figure e).

From the diagram determine c_2 c to be 24 000 m/s^2. This is the tangential acceleration of C to D. The angular acceleration of the link DC is then:

$\alpha_{(CD)}$ = 24000/0.16 = 150 000 rad/s^2 in a clockwise direction.

Note that although the link AB rotates at constant speed, the link CD has angular acceleration.

WORKED EXAMPLE No. 6

The same arrangement exists as shown for example 5 except that the link AB is decelerating at 8000 rad/s^2 (i.e. in an anticlockwise direction). Determine the acceleration of the link CD.

SOLUTION

The problem is essentially the same as example 5 except that a tangential acceleration now exists for point B relative to point A. This is found from

$$a_T = \alpha \times AB = 80000 \times 0.08 = 6400 \text{ m/s}^2$$

- The direction is for an anticlockwise tangent. This is vector b_1 b which is at right angles to a b_1 in the appropriate direction. The new acceleration diagram looks like this.

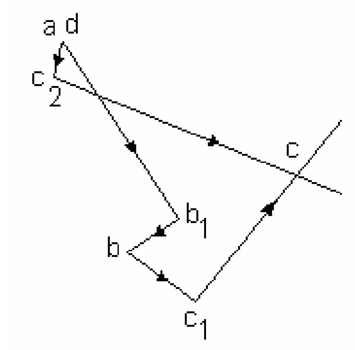


Figure 28

Scaling off the tangential acceleration c_2 c we get 19 300 m/s^2. Converting this into the angular acceleration we get

α = 19 300/0.16 = 120 625 rad/s^2 in a clockwise direction.

5. INERTIA FORCE

One of the reasons for finding the acceleration of links is to calculate the inertia force needed to accelerate or decelerate it. This is based on Newton's second law.

Force = mass x acceleration   F = M a

Torque = moment of inertia x angular acceleration   T = I α

WORKED EXAMPLE No.7

A horizontal single cylinder reciprocating engine has a crank OC of radius 40 mm and a connecting rod PC 140 mm long as shown.
The crank rotates at 3000 rev/min clockwise. For the configuration shown, determine the velocity and acceleration of the piston.

The sliding piston has a mass of 0.5 kg and a diameter of 80 mm. The gas pressure acting on it is 1.2 MPa at the moment shown. Calculate the effective turning moment acting on the crank. Assume that the connecting rod and crank has negligible inertia and friction.

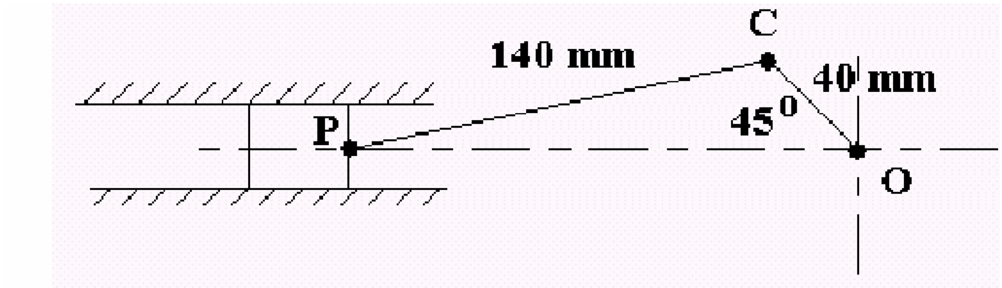


FIGURE: 29

SOLUTION

Draw the space diagram to scale.

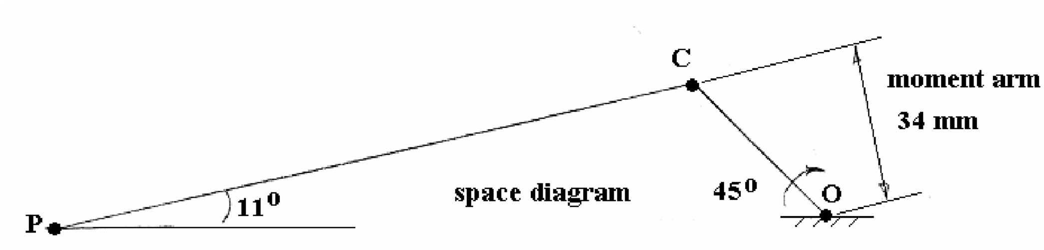


Figure 30

The moment arm should be scaled and found to be 34 mm (measured at right angles to the connecting rod PC.

Calculate the velocity of C relative to O.

$\omega = 2\pi N/60 = 2\pi \times 3000/60 = 314.16$ rad/s

$(VC)_O = \omega \times \text{radius} = 314.16 \times 0.04 = 12.57$ m/s

Draw the velocity diagram.

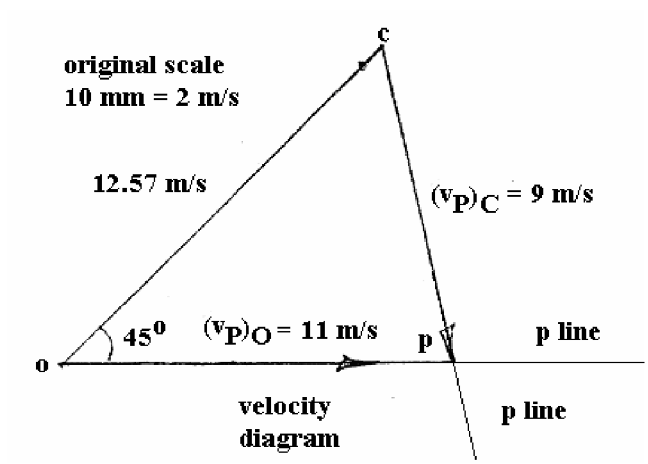


Figure 31

From the velocity diagram we find the velocity of the piston is 11 m/s.

Next calculate all the accelerations possible. Point C only has a radial acceleration towards O

Radial acceleration of C is v^2/radius = $12.57^2/0.04$ = 314.16 m/s2

Point P has radial and tangential acceleration relative to C. Tangential acceleration is unknown.

Radial acceleration = $(v_P)_C^2/CP = 9^2/0.14 = 578.57$ m/s^2

Now draw the acceleration diagram and it comes out like this.

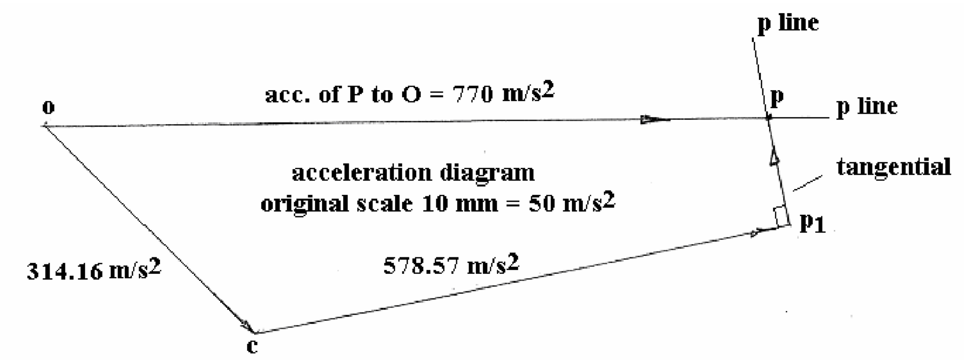


Figure 32

The acceleration of the piston is 770 m/s^2.

Now we can solve the forces.

Pressure force = p x area = $1.2 \times 10^6 \times \pi \times 0.08^2/4$ = 6032 N and this acts left to right. Inertia force acting on the piston = M a = 0.5 x 770 = 385 N and this must be provided by the pressure force so the difference is the force exerted on the connecting rod. Net Force = 6032 – 385 = 5647 N.

The connecting rod makes an angle of 11^o to the line of the force (angle scaled from space diagram). This must be resolved to find the force acting along the line of the connecting rod.

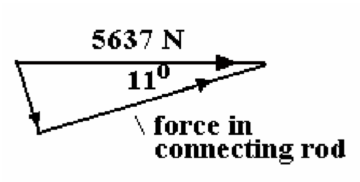


Figure 33

The force in the connecting rod is 5637 cos 11^o = 5543 N.

This acts at a radius of 34 mm from the centre of the crank so the torque provided by the crank is

T = 5545 x 0.034 = 188.5 N m.

UNIT III

KINEMATICS OF CAM

INTRODUCTION

CLASSIFICATION OF CAMS AND FOLLOWERS

- A cam is a mechanical element used to drive another element, called the follower, through a specified motion by direct contact.
- Cam-and-follower mechanisms are simple and inexpensive, have few moving parts, and occupy a very small space. Furthermore, follower motions having almost any desired characteristics are not difficult to design.
- For these reasons cam mechanisms are used extensively in modern machinery. The versatility and flexibility in the design of cam systems are among their more attractive features, yet this also leads to a wide variety of shapes and forms and the need for terminology to distinguish them.

Cams are classified according to their basic shapes. Figure 1 illustrates four different types of cams:

1. ***Plate cam, also called a disk cam or a radial cam***
2. ***Wedge cam***
3. ***Cylindrical cam or barrel cam***

Cam systems can also be classified according to the basic shape of the follower. Figure 2 shows plate cams acting with four different types of followers:

1. ***A knife-edge follower***
2. ***A flat-face follower***
3. ***A roller follower***
4. ***A spherical-face or curved-shoe follower***

- Notice that the follower face is usually chosen to have a simple geometric shape and the motion is achieved by proper design of the cant shape to mate with it.
- This is not always the case, and examples of inverse cams, where the output element is machined to a complex shape, can be found.
- Another method of classifying cams is according to the characteristic output motion allowed between the follower and the frame.
- Thus, some cams have reciprocating (translating) followers, as in Figs. 1 a through 1 d and Figs. 2a and 2b, while others have oscillating (rotating) followers, as in Fig. 1c and

Figs. 2c and 2d. Further classification of reciprocating followers distinguishes whether the centerline of the follower stem relative to the center of the cam is offset, as in Fig. 2a, or radial, as in Fig. 2b.

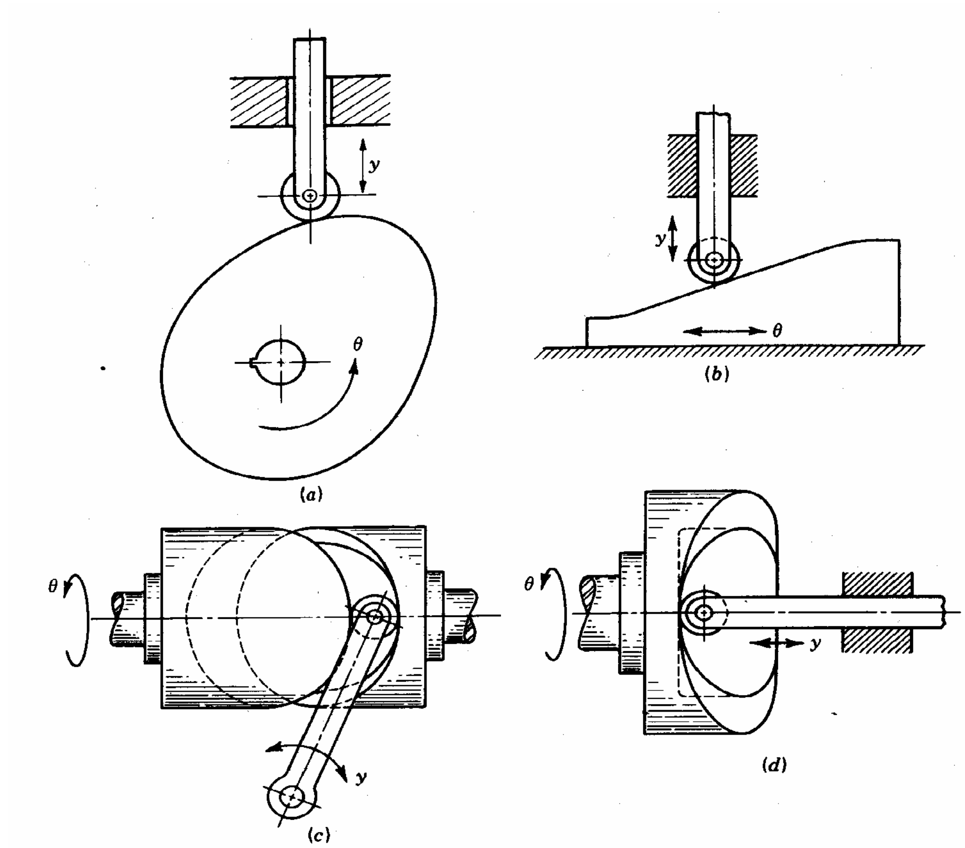


Figure: 1 Types of cams: (a) plate cam; (b) wedge cam; (c) barrel cam; (d) face cam

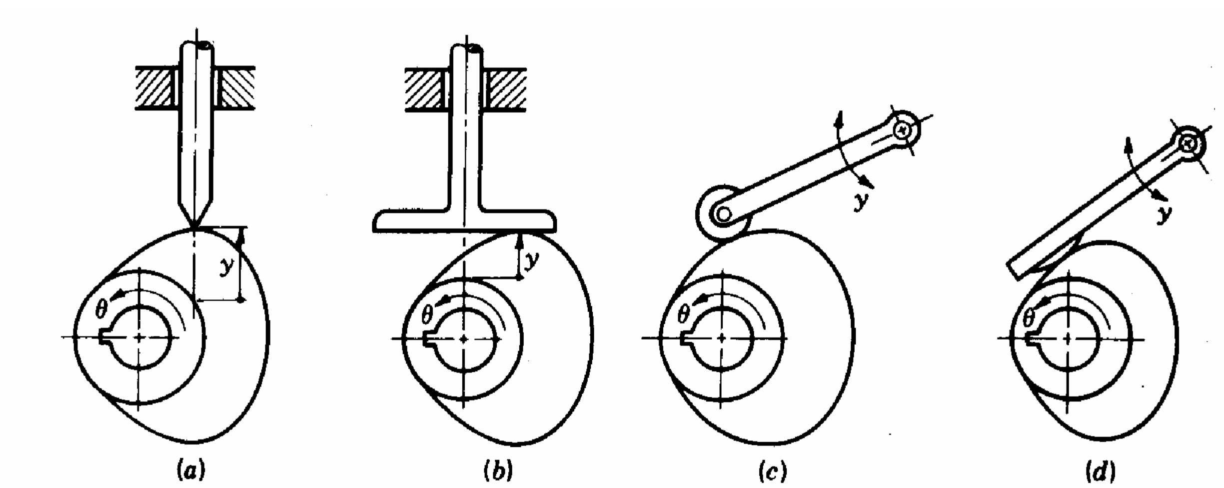


Fig: 2 Plate came with (a) an offset reciprocating knife edge follower; (b) reciprocating flat face follower ; (c) oscillating roller followers; (d) oscillating curved-shoe follower

- In all cam systems the designer must ensure that the follower maintains contact with the cam at all times. This can be done by depending on gravity, by the inclusion of a suitable spring, or by a mechanical constraint. In Fig.1c the follower is constrained by the groove.
- Figure 3a shows an example of a constant-breadth cam, where two contact points between the cam and the follower provide the constraint. Mechanical constraint can also be introduced by employing dual or conjugate cams in an arrangement like that illustrated in Fig. 3b. Here each cam has its own roller, but the rollers are mounted on a common follower.

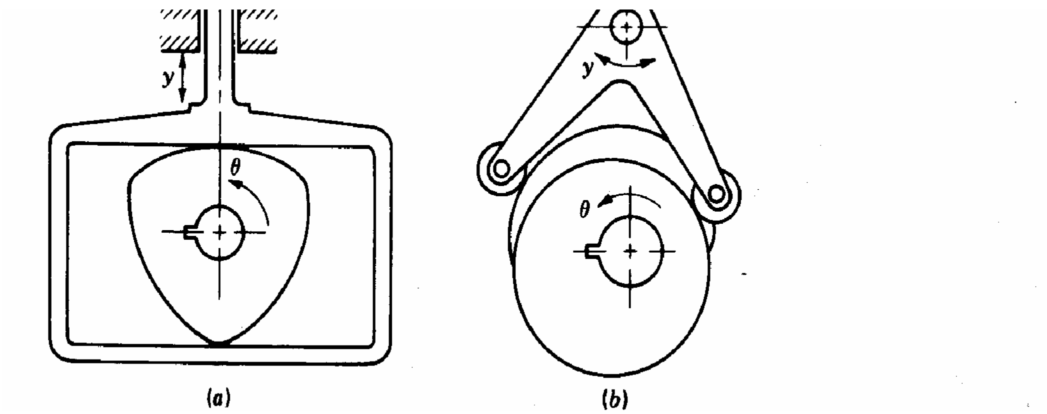


Fig 3: (a) a constant-breadth cam with a reciprocating flat face follower
(b) conjugate cams with an oscillating roller follower

DISPLACEMENT DIAGRAMS

- In spite of the wide variety of cam types used and their differences in form, they also have certain features in common which allow a systematic approach to their design. Usually a cam system is a single-degree-of-freedom device.
- It is driven by a known input motion, usually a shaft which rotates at constant speed, and it is intended to produce a certain desired output motion for the follower.
- In order to investigate the design of cams in general, we will denote the known input motion by θ (t) and the output motion by y. Reviewing Figs.1 to 3 will demonstrate the definitions of y and IJ for various types of cams. These figures also show that y is a transnational distance for a reciprocating follower but is an angle for an oscillating follower.
- During the rotation of the cam through one cycle of input motion, the follower executes a series of events as shown in graphical form in the displacement diagram of Fig. 3-4. In such a diagram the abscissa represents one cycle of the input motion θ (one revolution of the cam) and is drawn to any convenient scale.

- The ordinate represents the follower travel y and for a reciprocating follower is usually drawn at full scale to help in layout of the cam. On a displacement diagram it is possible to identify a portion of the graph called the rise, where the motion of the follower is away from the cam center.
- The maximum rise is called the lift. Periods during which the follower is at rest are referred to as dwells, and the return is the period in which the motion of the follower is toward the cam center.
- Many of the essential features of a displacement diagram, such as the total lift or the placement and duration of dwells, are usually dictated by the requirements of the application.
- There are, however, many possible follower motions, which can be used for the rise and return, and some are preferable to others depending on the situation.

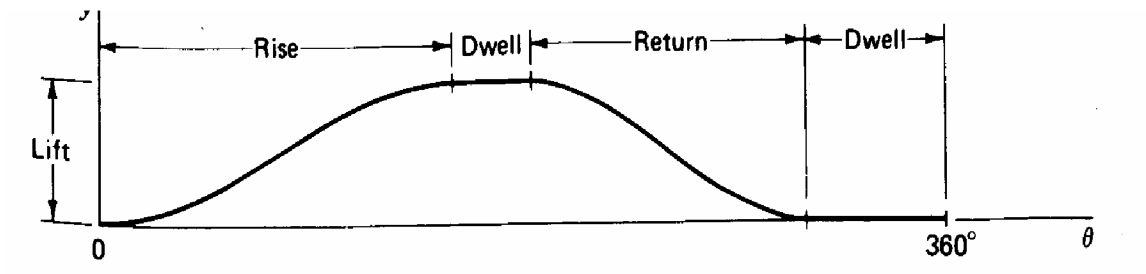


Figure 4: Displacement diagram for a cam

- One of the key steps in the design of a cam is the choice of suitable forms for these motions.
- Once the motions have been chosen, that is, once the exact relationship is set between the input θ and the output y, the displacement diagram can be constructed precisely and is a graphical representation of the functional relationship

$$y = y(\theta)$$

- This equation has stored in it the exact nature of the shape of the final cam, the necessary information for its layout and manufacture, and also the important characteristics, which determine the quality of its dynamic performance.
- Before looking further at these topics, however, we will display graphical methods of constructing the displacement diagrams for various rise and return motions.
- The displacement diagram for uniform motion is a straight line with a constant slope. Thus, for constant input speed, the velocity of the follower is also constant.

- This motion is not useful for the full lift because of the corners produced at the boundaries with other sections of the displacement diagram. It is often used, however, between other curve sections, thus eliminating the corners.
- The displacement diagram for a modified uniform motion is illustrated in Fig. 5a. The central portion of the diagram, subtended by the cam angle β_2 and the lift L_2 is uniform motion.
- The ends, angles $\beta_1$and β_3 and corresponding lifts L_1 and L_3 are shaped to deliver parabolic motion to the follower. Soon we shall learn that this produces constant acceleration. The diagram shows how to match the slopes of the parabolic motion with that of the uniform motion.
- With $\beta_{1,}$ $\beta_2$and β_3 and the total lift L known, the individual lifts $L_{1,}$ L_2 and L_3 are found by locating the midpoints of the β_1, and β_3 sections and constructing a straight line as shown.
- Figure 5b illustrates a graphical construction for a parabola to be fit within a given rectangular boundary defined by L_1 and β_1.
- The abscissa and ordinate are first divided into a convenient but equal number of divisions and numbered as shown. The construction of each point of the parabola then follows that shown by dashed lines for point 3.

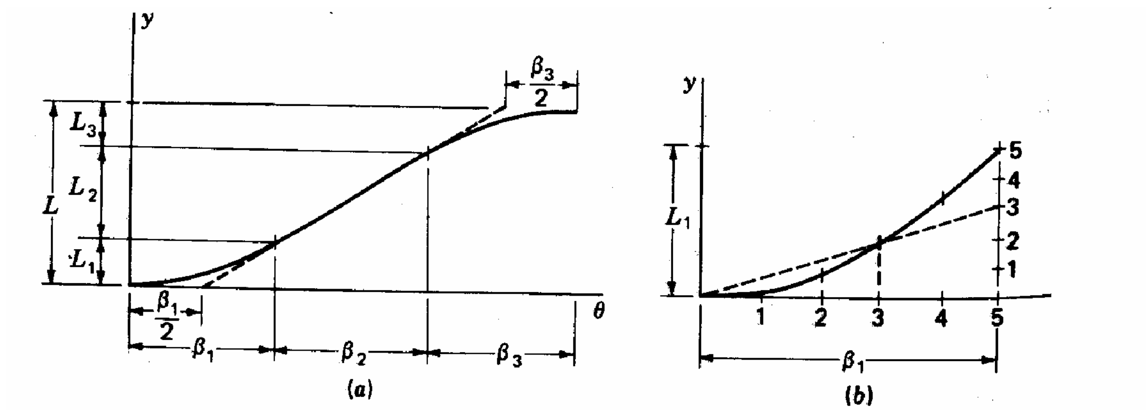


Figure 5: Parabolic motion displacement diagram: (a) interfaces with uniform motion; (b) graphical construction

- In the graphical layout of an actual cam, a great many divisions must be employed to obtain good accuracy. At the same time, the drawing is made to a large scale, perhaps 10 times size.
- However, for clarity in reading, the figures in this chapter are shown with a minimum number of points to define the curves and illustrate the graphical techniques.
- The displacement diagram for simple harmonic motion is shown in Fig.6. The graphical construction makes use of a semicircle having a diameter equal to the rise L.

- The semicircle and abscissa are divided into an equal number of parts and the construction then follows that shown by dashed lines for point 2.

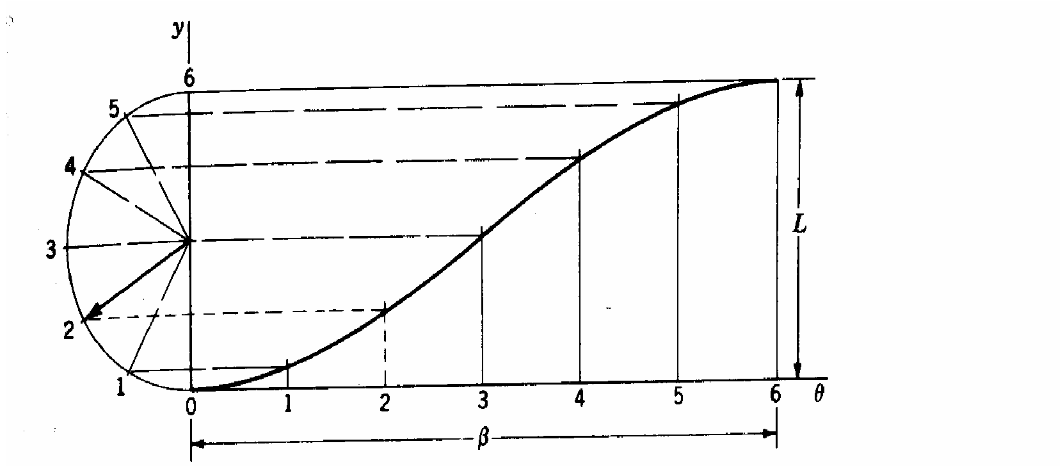


Figure 6: simple harmonic motion displacement diagram; graphical construction

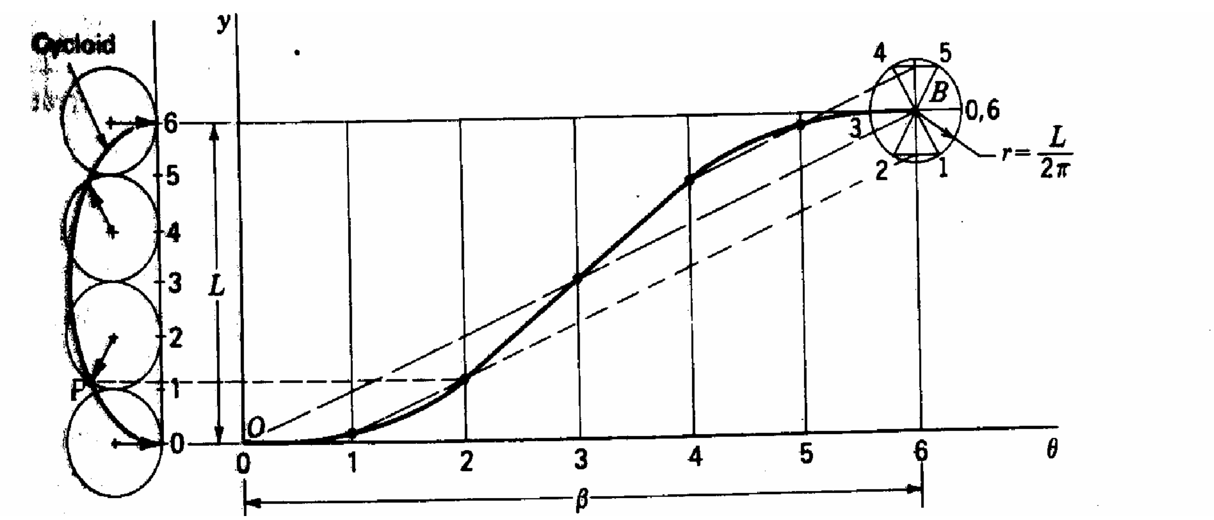


Figure 7: Cycloidal ion displacement diagram; graphical construction

- Cycloidal motion obtains its name from the geometric curve called a cycloid. As shown in the left of Fig. 7, a circle of radius L /2π, where L is the total rise, will make exactly one revolution when rolling along the ordinate from the origin to y = L. A point P of the circle, originally located at the origin, traces a cycloid as shown.
- If the circle rolls without slip at a constant rate, the graph of the point's vertical position y versus time gives the displacement diagram shown at the right of the figure. We find it much more convenient for graphical purposes to draw the circle only once, using point B as a center.
- After dividing the circle and the abscissa into an equal number of parts and numbering them as shown, we project each point of the circle horizontally until it intersects the

ordinate; next, from the ordinate, we project parallel to the diagonal OB to obtain the corresponding point on the displacement diagram.

GRAPHICAL LAYOUT OF CAM PROFILES

- Let us now examine the problem of determining the exact shape of a cam surface required to deliver a specified follower motion.
- We assume here that the required motion has been completely determined-graphically, analytically, or numerically-as discussed in later sections.
- Thus a complete displacement diagram can be drawn to scale for the entire cam rotation. The problem now is to layout the proper cam shape to achieve the follower motion represented by this displacement diagram.
- We will illustrate for the case of a plate cam as shown in Fig. 8. Let us first note some additional nomenclature shown on this figure.
- The trace point is a theoretical point of the follower; it corresponds to the point of a fictitious knife-edge follower. It is chosen at the center of a roller follower or on the surface of a flat-face follower.
- The pitch curve is the locus generated by the trace point as the follower moves relative to the cam. For a knife-edge follower the pitch curve and cam surface are identical. For a roller follower they are separated by the radius of the roller.
- The prime circle is the smallest circle that can be drawn with center at. The cam rotation axis and tangent to the pitch curve. The radius of this circle is R_o.
- The base circle is the smallest circle centered on the cam rotation axis and tangent to the cam surface. For a roller follower it is smaller than the prime circle by the radius of the roller, and for a flat-face follower it is identical with the prime circle.
- In constructing the cam profile, we employ the principle of kinematics inversion, imagining the cam to be stationary and allowing the follower to rotate opposite to the direction of cam rotation.
- As shown in Fig. 8, we divide the prime circle into a number of segments and assign station numbers to the boundaries of these segments.
- Dividing the displacement-diagram abscissa into corresponding segments, we can then transfer distances, by means of dividers, from the displacement diagram directly onto the cam layout to locate the corresponding positions of the trace point.

- A smooth curve through these points is the pitch curve. For the case of a roller follower, as in this example, we simply draw the roller in its proper position at each station and then construct the Cam profile as a smooth curve tangent to all these roller positions.

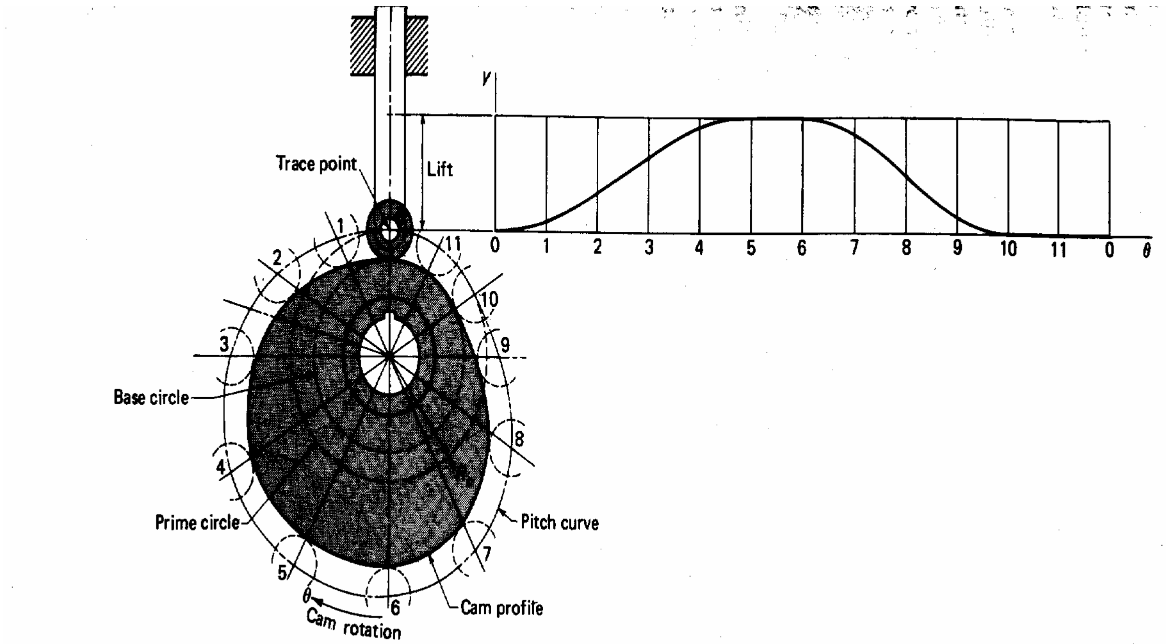


Figure 8: Cam nomenclature

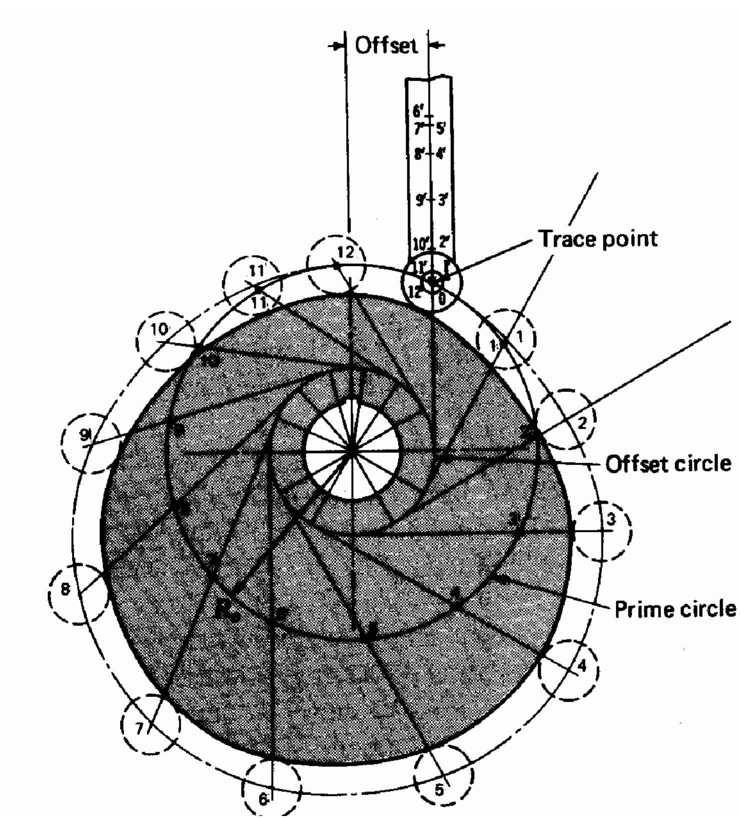


Figure 9: Graphical layout of a plate cam profile with an offset reciprocating roller follower

- Figure 9 shows how the method of construction must be modified for an offset roller follower. We begin by constructing an offset circle, using a radius equal to the amount of offset.
- After identifying station numbers around the prime circle, the centerline of the follower is constructed for each station making it tangent to the offset circle.
- The roller centers for each station are now established by transferring distances from the displacement diagram directly to these follower centerlines, always measuring outward from the prime circle.
- An alternative procedure is to identify the points 0', 1', 2', etc., on a single follower centerline and then to rotate them about the cam center to the corresponding follower centerline positions.
- In either case the roller circles can be drawn next and a smooth curve tangent to all roller circles is the required cam profile.

DERIVATIVES OF THE FOLLOWER MOTION

We have seen that the displacement diagram is plotted with the follower motion y as the ordinate and the cam input motion θ as the abscissa no matter what the type of the cam or follower. The displacement diagram is therefore a graph representing some mathematical function relating the input and output motions f the cam system. In general terms, this relationship, is

$$Y = y(\theta) \rightarrow (3.1)$$

If we wished to take the trouble, we could plot additional graphs representing derivatives of y with respect to θ. The first derivative we will denote as y':

$$y^{|}(\theta) = \frac{dy}{d\theta} \rightarrow (3.2)$$

It represents the slope of the displacement diagram at each position θ. This derivative, although it may now seem of little practical value, is a measure of “steepness" of the displacement diagram.

- We will find in later sections that it is closely related to the mechanical advantage of the cam system and manifests itself in such things as pressure angle. If we consider a wedge cam (Fig.1 b) with a knife-edge follower, the displacement diagram itself is of he same shape as the corresponding cam.

- Here we can begin to visualize that difficulties will occur if the cam is too steep, that is, if y' has too high a value.

The second derivative of y with respect to θ is also significant. It is denoted here as y":

$$y^{||}(\theta) = \frac{d^2 y}{d\theta^2} \rightarrow (3.3)$$

Although it is not quite as easy to visualize, this derivative is very closely related to the radius of curvature of the cam at various points along its profile.

- Since there is an inverse relationship, as y" becomes very large, the radius of curvature becomes very small; if y" becomes infinite, the cam profile at that, position becomes pointed, a highly unsatisfactory condition from the point of view of contact stresses between the cam and follower surfaces.

The next derivative can also be plotted if desired:

$$y^{|||}(\theta) = \frac{d^3 y}{d\theta^3} \rightarrow (3.4)$$

- Although it is not easy to describe geometrically, this is the rate of the change of y", and we will see below that this derivative should also be controlled when choosing the detailed shape of the displacement diagram.

Example 3.1 Derive equations to describe the displacement diagram of a cam which rises with parabolic motion from a dwell to another dwell such that the total lift is L and the total cam rotation angle is β. Plot the displacement diagram and its first three derivatives with respect to cam rotation.

Solution As illustrated in Fig. 5a, two parabolas will be required, meeting at an Inflection point taken here at midrange. For the first half of the motion we choose the general equation of a parabola,

$$y = A\theta^2 + B\theta + C \quad \text{(a)}$$

which has derivatives

$$y' = 2A\theta + B \quad \text{(b)}$$

$$y'' = 2A \quad \text{(c)}$$

$$y''' = 0 \quad \text{(d)}$$

To match the position and slope with those of the preceding dwell properly, at $\theta = 0$ we have $y(0) = y'(0) = 0$. Thus, Eqs. (a) and (b) show that $B = C = 0$. Looking next at the inflection point, at $\theta = \beta/2$ we want $y = L/2$; Eq. (a) yields

$$A = \frac{2L}{\beta^2}$$

Thus, for the first half of the parabolic motion, the equations are

$$y = 2L\left(\frac{\theta}{\beta}\right)^2 \tag{5-5}$$

$$y' = \frac{4L}{\beta}\frac{\theta}{\beta} \tag{5-6}$$

$$y'' = \frac{4L}{\beta^2} \tag{5-7}$$

$$y''' = 0 \tag{5-8}$$

The maximum slope occurs at the inflection point, where $\theta = \beta/2$. Its value is

$$y'_{\max} = \frac{2L}{\beta} \tag{5-9}$$

For the second half of the motion we return to the general equations (a) through (d) for a parabola. Substituting the conditions that at $\theta = \beta$, $y = L$, and $y' = 0$, we have

$$L = A\beta^2 + B\beta + c \tag{e}$$

$$0 = 2A\beta + B \tag{f}$$

Since the slope must match that of the first parabola at $\theta = \beta/2$, we have, from Eqs. (5-9) and (b),

Solving Eqs. (e) through (g) simultaneously gives

$$A = -\frac{2L}{\beta^2} \qquad B = \frac{4L}{\beta} \qquad C = -L$$

When these constants are substituted into the general forms, we obtain the

From the general equation of the displacement diagram,

$$y = y(\theta) \qquad \theta = \theta(t)$$

We can therefore differentiate to find the time derivatives of the follower motion. The velocity of the follower, for example, is given by

$$\dot{y} = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt}$$

$$\dot{y} = y'\omega \tag{5-14}$$

Similarly, the acceleration and jerk of the follower are given by

$$\ddot{y} = \frac{d^2y}{dt^2} = y''\omega^2 + y'\alpha \tag{5-15}$$

and

$$\dddot{y} = \frac{d^3y}{dt^3} = y'''\omega^3 + 3y''\omega\alpha + y'\dot{\alpha} \tag{5-16}$$

When the camshaft speed is constant, these reduce to

$$\dot{y} = y'\omega \qquad \ddot{y} = y''\omega^2 \qquad \dddot{y} = y'''\omega^3 \tag{5-17}$$

- For this reason, it has become somewhat common to refer to graphs of the kinematics derivatives y', y", and y^{iii}, such as those shown in Fig. 12, as the "velocity," "acceleration," and "jerk" curves for a given motion.
- They would be appropriate names for a constant-speed cam only, and then only when scaled by ω, ω^2, and ω^3, respectively. However, it is helpful to use these names for the derivatives when considering the physical implications of a certain choice of displacement diagram. For the parabolic motion of Fig.12, for example, the "velocity" of the follower rises linearly to a maximum and then decreases to zero.
- The "acceleration" of the follower is zero during the initial dwell and changes abruptly to a constant positive value upon beginning the rise. There are two more abrupt changes in "acceleration" of the follower, at the midpoint and end of the rise. At each of the abrupt changes of "acceleration," the "jerk" of the follower becomes infinite.

HIGH-SPEED CAMS

- Continuing with our discussion of parabolic motion, let us consider briefly the implications of the "acceleration" curve of Fig.10 on the dynamic performance of the cam system.
- Any real follower must, of course, have some mass and, when multiplied by acceleration, will exert an inertia force. Therefore, the "acceleration" curve of Fig. 10 can also be

thought of as indicating the inertia force of the follower, which, in turn, must be felt at the follower bearings and at the contact point with the cam surface.

- An "acceleration" curve with abrupt changes, such as parabolic motion, will exert abruptly changing contact stresses at the bearings and on the cam surface and lead to noise, surface wear, and eventual failure.

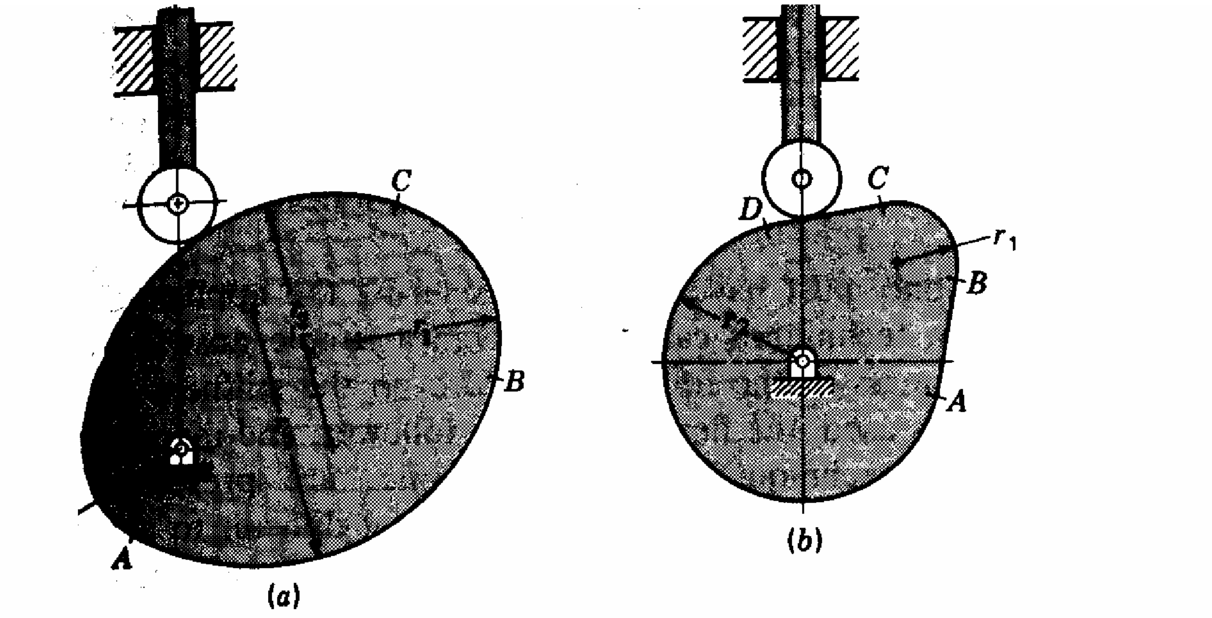


Figure 10: (a) Circle-arc cam (b) Tangent cam

- Thus it is very important in choosing a displacement diagram to ensure that the first and second derivatives, the "velocity" and "acceleration" curves, are continuous, that is, that they contain no step changes.
- Sometimes in low-speed applications compromises are made with the velocity and acceleration relationships. It is sometimes simpler to employ a reverse procedure and design the cam shapes first, obtaining the displacement diagram as a second step.
- Such cams are often composed of some combination of curves such as straight lines and circular arcs which are readily produced by machine tools. Two examples are the circle-arc cam and the tangent cam of Fig.10. The design approach is by iteration.
- A trial cam is designed and its kinematics characteristics computed. The process is then repeated until a cam with the desired characteristics is obtained. Points A, B, C, and D of the circle-arc and tangent cams are points of tangency or blending points. It is worth noting, as with the parabolic-motion example above, that the acceleration changes abruptly at each of the blending points because of the instantaneous change in radius of curvature.

- v Although cams with discontinuous acceleration characteristics are some- times found in low-speed applications, such cams are certain to exhibit major problems as the speed is raised.
- v For any high-speed cam application, it is extremely important that not only the displacement and "velocity" curves but also the "acceleration" curve be made continuous for the entire motion cycle. No discontinuities should be allowed at the boundaries of different sections of the cam.
- v How high a speed one must have before considering the application to require high-speed design techniques cannot be given a simple answer.
- v This depends not only on the mass of the follower but also on the stiffness of the return spring, the materials used, the flexibility of the follower, and many other factors. Still, with the methods presented below, it is not difficult to achieve continuous derivative displacement diagrams. Therefore it is recommended that this be done as standard practice.
- v Parabolic-motion cams are no easier to manufacture than Cycloidal motion cams, for example, and there is no good reason for their use. The circle-arc and tangent cams are easier to produce, but with modern machining methods cutting of more complex cam shapes is not expensive.

STANDARD CAM MOTIONS

- v Details of the equations for parabolic motion and its derivatives have already been seen in the previous sections.
- v The purpose of this section is to present equations for a number of standard types of displacement curves which can be used to address most high-speed cam-motion requirements.

The displacement diagram and its derivatives for simple harmonic rise motion are shown in Fig. 11. The equations are

$$y = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right) \qquad y'' = \frac{\pi^2 L}{2\beta^2}\cos\frac{\pi\theta}{\beta}$$

$$y' = \frac{\pi L}{2\beta}\sin\frac{\pi\theta}{\beta} \qquad y''' = -\frac{\pi^3 L}{2\beta^3}\sin\frac{\pi\theta}{\beta}$$

Unlike parabolic motion, simple harmonic motion shows no discontinuity at the inflection point.

UNIT 4

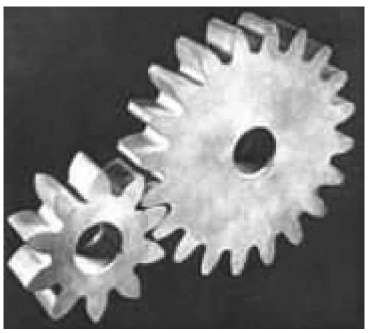
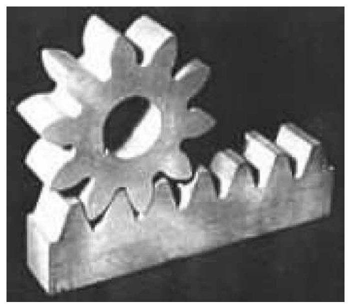
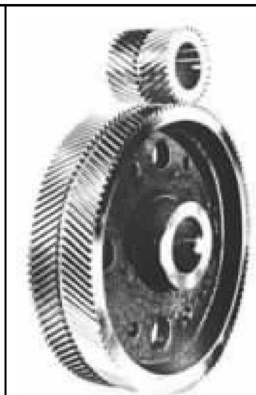
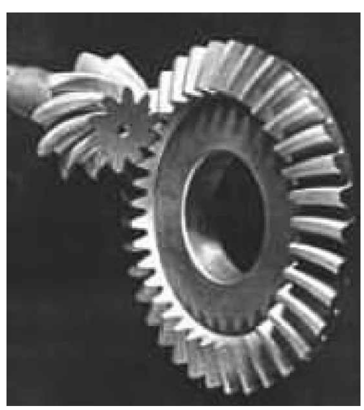
GEARS

SYLLUBUS

"Spur gear Terminology and definitions-Fundamental Law of toothed gearing and involute gearing-Inter changeable gears-gear tooth action – Terminology - Interference and undercutting-Non standard gear teeth- Helical, Bevel, Worm, Rack and Pinion gears (Basics only)-Gear trains-Parallel axis gear trains-Epicyclic gear trains-Differentials"

SPUR GEARS

Gears are used to transmit power between shafts rotating usually at different speeds. Some of the many types of gears are illustrated below :

		
1. A pair of ***spur*** gears for mounting on parallel shafts. The 10 teeth of the smaller ***pinion*** and the 20 teeth of the ***wheel*** lie parallel to the shaft axes	2. A ***rack*** and pinion. The straight rack translates rectilinearly and may be regarded as part of a wheel of infinite diameter	3. Like spur gears ***helical*** gears connect parallel shafts, however the teeth are not parallel to the shaft axes but lie along helices about the axes
		
4. Straight ***bevel*** gears for shafts whose axes intersect	***5. Hypoid*** gears - one of a number of gear types for offset shafts.	6. A ***worm*** and worm wheel gives a large Speed ratio.

TERMINOLOGY OF SPUR GEAR:

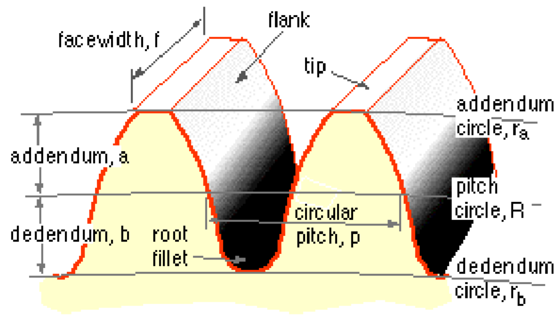


Fig. 1: Terminology of spur gear

- A pair of meshing gears is a power transformer, a coupler or interface which marries the speed and torque characteristics of a power source and a power sink (load).
- A single pair may be inadequate for certain sources and loads, in which case more complex combinations such as the above gearbox, known as ***gear trains,*** are necessary.
- In the vast majority of applications such a device acts as a **speed reducer** in which the power source drives the device through the high speed low torque input shaft, while power is fed from the device to the load through the low speed high torque output shaft.
- Speed reducers are much more common than speed -up drives not so much because they reduce speed, but rather because they amplify torque.
- Thus gears are used to accelerate a car from rest, not to provide the initial low speeds (which could be accomplished by easing up on the accelerator pedal) but to increase the torque at the wheels which is necessary to accelerate the vehicle.

These notes will consider the following aspects of spur gearing :-

- Overall kinetics of a gear pair (for cases only of steady speeds and loads)
- Tooth geometry requirements for a constant velocity ratio (eg. size and conjugate action)
- Detailed geometry of the involute tooth and meshing gears
- The consequences of power transfer on the fatigue life of the components, and hence
- The essentials of gear design.
- Some of the main features of spur gear teeth are illustrated. The teeth extend from the root, or ***dedendum*** cylinder (or colloquially, *"circle"*) to the tip, or ***addendum*** circle: both these circles can be measured.

- The useful portion of the tooth is the ***flank*** (or face), it is this surface which contacts the mating gear.
- The ***fillet*** in the root region is kinematically irrelevant since there is no contact there, but it is important insofar as fatigue is concerned.

CONDITION FOR CONSTANT VELOCITY RATIO OF TOOTHED WHEELS (LAW OF GEARING)

- Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 2.
- Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure. Let TT be the common tangentand MN be the common normal to the curves at the point of contact Q.
- From the centres $O_1$and O_2 draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.
- Let v_1 and v_2 be the velocities of the point Q od the wheels I and 2 respectively. If the teeth are to remain in contact, then the components of these ye locities akng the common normal MN must be equal.

$$v_1 \cos \alpha = v_2 \cos \beta$$

$$(\omega_1 \times O_1Q) \cos \alpha = (\omega_2 \times O_2Q) \cos \beta$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

- From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

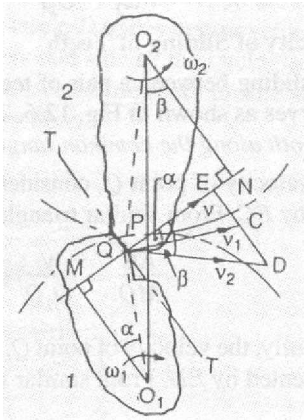


Fig. 2. Law of gearing.

- Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels.
- In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is the fundaniental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

INTERFERENCE IN INVOLUTE GEARS

- Fig. 3 shows a pinion with centre O_1 in mesh with wheel or gear with centre O_2.
- MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.

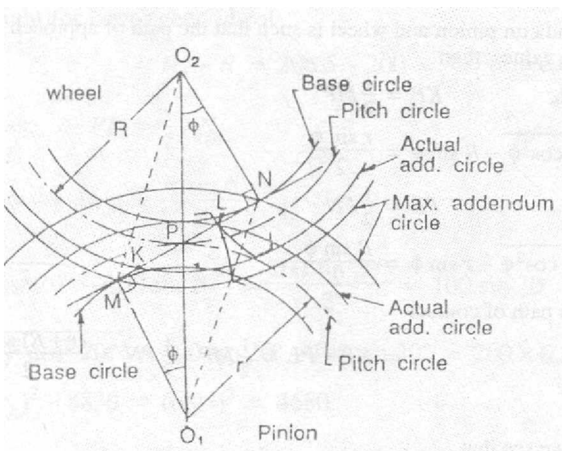


Fig.3. Interference in involute gears.

- A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N the point of contact L will move from L to N.

- v When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel.
- v This effect is known as !nterference, and occurs when the teeth are being cut. In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.
- v Similarly, if the radius of the addendum circle of the wht I increases beyond O_2M then the tip of tooth on wheel will cause interference with the tooth on pinion.
- v The points M and N are called interference points. Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.
- v From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, interference may only be prevented (f the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.

Maximum length of path of contact,

$$MN = MP + PN = r \sin\Phi + R \sin\Phi = (r + R) \sin\Phi$$

OVERALL KINETICS OF A GEAR PAIR

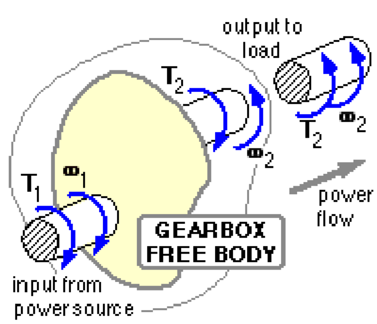


Fig : 4 Overall kinetics of a gear pair

- Analysis of gears follows along familiar lines in that we consider kinetics of the overall assembly first, before examining internal details such as individual gear teeth. The free body of a typical single stage gearbox is shown.
- The power source applies the torque T_1 to the input shaft, driving it at speed ω_1 in the sense of the torque (clockwise here).

- For a single pair of gears the output shaft rotates at speed ω_2 in the opposite sense to the input shaft, and the torque T_2 supplied by the gearbox drives the load in the sense of ω_2.
- The reaction to this latter torque is shown on the free body of the gearbox - apparently the output torque T_2 must act on the gearbox ***in the same sense*** as that of the input torque T_1.
- The gears appear in more detail in Fig 5(**i**) below. O_1 and O_2 are the centres of the pinion and wheel respectively. We may regard the gears as equivalent ***pitch cylinders*** which roll together without slip - the requirements for preventing slip due to the ***positive drive*** provided by the meshing teeth is examined below.
- Unlike the addendum and dedendum cylinders, pitch cylinders cannot be measured directly; they are notional and must be inferred from other measurements.

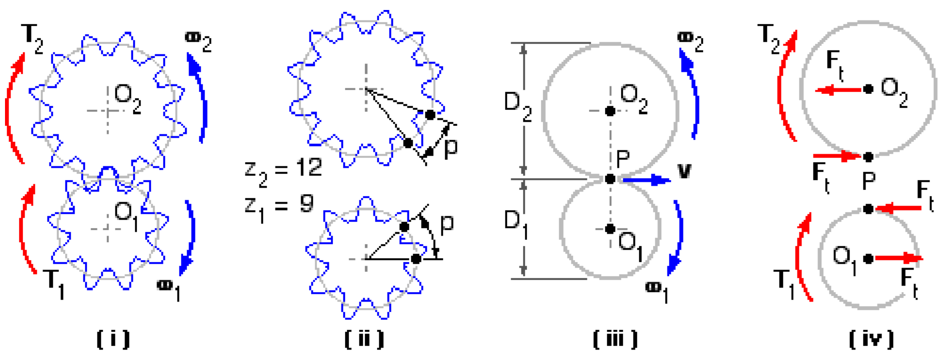


Fig: 5 spur gear assembly

- One essential for correct meshing of the gears is that the ***size*** of the teeth on the pinion is the same as the size of teeth on the wheel.
- One measure of size is the ***circular pitch,*** p, the distance between adjacent teeth around the pitch circle Fig 5 (**ii**); thus $p = \pi D/z$ where z is the number of teeth on a gear of pitch diameter D.
- The SI measure of size is the ***module,*** $m = p/\pi$ - which should not be confused with the SI abbreviation for metre. So the geometry of pinion 1 and wheel 2 must be such that :

$$D_1 / z_1 = D_2 / z_2 = p / \pi = m$$

- That is the module must be common to both gears. For the rack illustrated above, both the diameter and tooth number tend to infinity, but their quotient remains the finite module.

The pitch circles contact one another at the ***pitch point,*** P Fig 5(**iii**), which is also notional. Since the positive drive precludes slip between the pitch cylinders, the pinion's pitch line velocity, v, must be identical to the wheel's pitch line velocity :

$$v \;=\; \omega_1 R_1 \;=\; \omega_2 R_2 \qquad ; \qquad \text{where pitch circle radius } \; R = D/2$$

- Separate free bodies of pinion and wheel appear in Fig 5(**iv**).F_t is the tangential component of action -reaction at the pitch point due to contact between the gears.
- The corresponding radial component plays no part in power transfer and is therefore not shown on the bodies. Ideal gears only are considered initially, so the friction force due to sliding contact is omitted also.

The free bodies show that the magnitude of the shaft reactions must be F_t, and that for equilibrium :

$$F_t \;=\; T_1 / R_1 \;=\; T_2 / R_2 \qquad \text{in the absence of friction.}$$

The preceding concepts may be combined conveniently into :-

$$1) \;\; \omega_1 / \omega_2 \;=\; T_2 / T_1 \;=\; D_2 / D_1 \;=\; z_2 / z_1 \qquad ; \qquad D = mz$$

v That is, gears reduce speed and amplify torque in proportion to their teeth numbers. In practice, rotational speed is described by N (rev/min or Hz) rather than by ω (rad/s).

v There exists a host of shapes which ensure conjugacy - indeed it is possible, within certain restrictions, to arbitrarily choose the shape of one body then determine the shape of the second necessary for conjugacy.

v But by far the most common gear geometry which satisfies conjugacy is based on ***the involute,*** in which case both gears are similar in form, and the contact point's locus is a simple straight line - the ***line of action.***

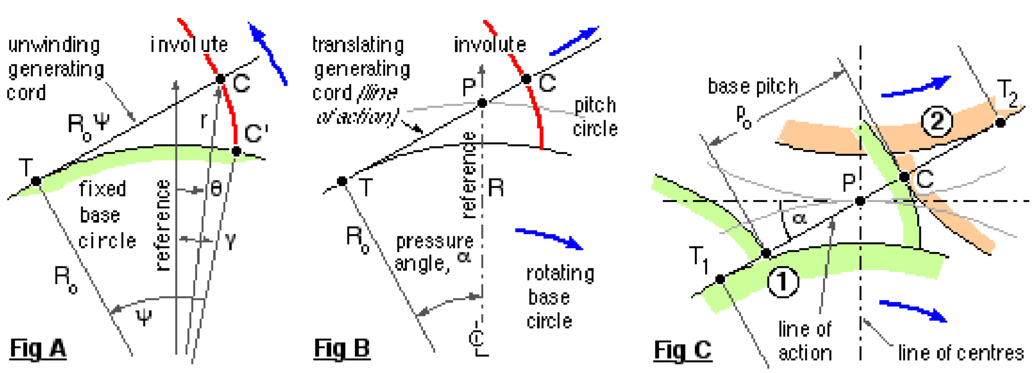


Fig : 6 The involute tooth

- One method of generating an involute is shown in Fig 6. A. A generating cord, in which there is a knot C, is wrapped around a fixed cylinder - the ***base cylinder*** (idiomatically circle) of radius R_o.
- When the taut cord is subsequently unwound as shown in this **animation**, the knot traces out an involute whose polar coordinates may be expressed implicitly in terms of the variable generating angle ψ, reckoned from the radius through the initial knot position, C'.
- The coordinate origin is taken at the circle centre, O, with a fixed reference direction defined at some constant angle γ, also from the initial radius.

The tangent, TC, is normal to the involute at C, and since the tangent length TC is equal to the arc length TC', the polar coordinates of C (r, θ) are :-

$$2) \quad r = R_o \sqrt{(1 + \psi^2)} \quad ; \quad \theta = \gamma - \psi + \arctan \psi$$

- In order to see how the involute leads to gear teeth and conjugate action, we place a slightly different interpretation on the above model. The cord is wrapped around the base cylinder which in Fig 6.B is now free to rotate about its centre as the cord is pulled off in a fixed direction.
- This fixed cord direction forms the line of action, tangent to the base cylinder at the fixed point T, and clearly satisfies conjugacy by cutting the fixed reference at the fixed pitch point P through which the pitch cylinder passes.
- The line of action is inclined to the pitch point tangent at the ***pressure angle,*** α. The knot C always moves along the line of action, tracing out an involute with respect to the rotating cylinder.

The relation between the base and pitch circle radii is evidently :-

$$3) \quad R_o = R \cos \alpha$$

- Extending this to two cylinders - representing meshing gears, 1 & 2 Fig C - the taut cord winds off one base cylinder and onto the other to form the line of action inclined at the pressure angle α.
- The knot, C, on the mating involutes coincides with the contact point and moves along the line of action as the gears and base cylinders rotate. The pitch cylinders extend to the pitch point P situated at the intersection of the lines of action and of centres.
- Evidently the distance between the cylinders does not affect the speed ratio since the base cylinder diameters are fixed.

The distance between knots - ie. between tooth flanks along the line of action, Fig C - is the ***base pitch,*** p_o, given by :-

$$4)\ p_o = \pi D_o / z = p \cos\alpha = \pi m \cos\alpha \quad \text{. . . . . from (\textbf{1})}$$

- For continuous motion transfer, at least two pairs of teeth must be in contact as one of the pairs comes into or leaves mesh. The teeth in Fig.6.C are truncated in practice to permit rotation.
- Involute generation by knotted cord is all very well conceptually, but hardly practicable as a basis for manufacturing.
- Only one of the many methods of gear manufacture is considered here - the **rack generation** technique is fundamental to the understanding of gear behaviour.

CONJUGATE TOOTH ACTION

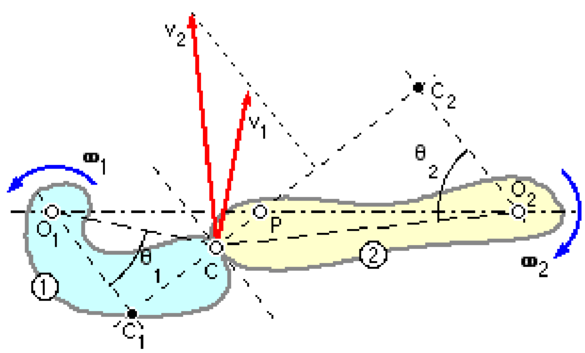


Fig : 7 Conjugate tooth action

- We have seen that one essential for correctly meshing gears is that the size of the teeth (the module) must be the same for the two gears.
- We now examine another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behaviour of the contacting surfaces (ie. the teeth flanks) is known as ***conjugate action.***
- Consider the two rigid bodies 1 and 2 which rotate about fixed centres, O, with angular velocities ω. The bodies touch at the contact point, C, through which the common tangent and normal are drawn.
- The absolute velocity v of the contact point reckoned as a point on either body, is perpendicular to the radius from that body's centre O to the contact point.
- For the bodies to remain in contact, there must be no component of relative motion along the common normal, so that from the velocity triangles :-

$$v_2 \cos\theta_2 \;=\; v_1 \cos\theta_1 \qquad \text{where} \quad v_1 = \omega_1 \,.\, O_1C \quad ; \qquad v_2 = \omega_2 \,.\, O_2C$$

Note that the tangential components of velocity are generally different, so sliding must occur. For the speed ratio to be constant therefore, from the above and similar triangles :-

5) $\omega_2 / \omega_1 \;=\; v_2 \,.\, O_1C / v_1 \,.\, O_2C \;=\; O_1C.\cos\theta_1 / O_2C.\cos\theta_2$

$= \; O_1 C_1 / O_2 C_2 \;=\; O_1 P \,/\, O_2 P$ ie. this ratio also must be constant.

This indicates that, since the centres are fixed, ***the point P is fixed*** too.

In general therefore, whatever the shapes of the bodies, the contact point C will move along some locus as rotation proceeds; but if the action is to be conjugate then the body geometry must be such that the common normal at the contact point passes always through one unique point lying on the line of centres - this point is the pitch point referred to above, and the pitch circles' radii are $O_1 P$ and $O_2 P$.

- There exists a host of shapes which ensure conjugacy - indeed it is possible, within certain restrictions, to arbitrarily choose the shape of one body then determine the shape of the second necessary for conjugacy.
- But by far the most common gear geometry which satisfies conjugacy is based on ***the involute,*** in which case both gears are similar in form, and the contact point's locus is a simple straight line - the ***line of action.***

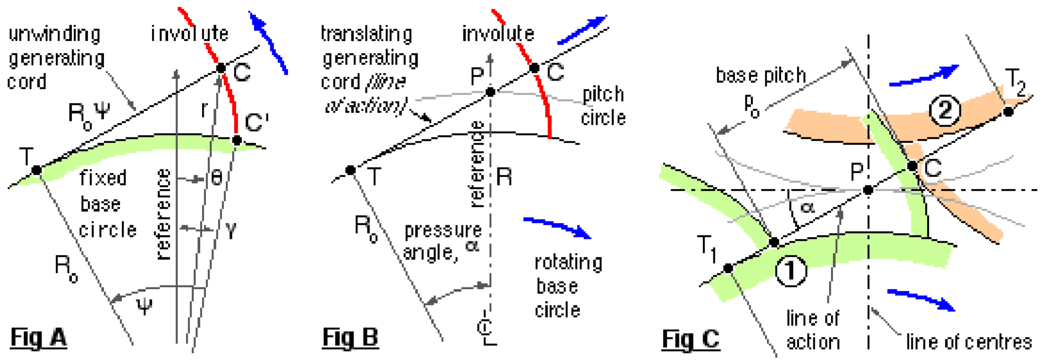


Fig : 8 **The involute tooth**

- One method of generating an involute is shown in Fig 8 .A. A generating cord, in which there is a knot C, is wrapped around a fixed cylinder - the ***base cylinder*** (idiomatically circle) of radius.R_o.
- When the taut cord is subsequently unwound as shown in this **animation**, the knot traces out an involute whose polar coordinates may be expressed implicitly in terms of

the variable generating angle ψ, reckoned from the radius through the initial knot position, C'.

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- The cord is wrapped around the base cylinder which in Fig.8.B is now free to rotate about its centre as the cord is pulled off in a fixed direction.
- This fixed cord direction forms the line of action, tangent to the base cylinder at the fixed point T, and clearly satisfies conjugacy by cutting the fixed reference at the fixed pitch point P through which the pitch cylinder passes.
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- The knot, C, on the mating involutes coincides with the contact point and moves along the line of action as the gears and base cylinders rotate.
- The pitch cylinders extend to the pitch point P situated at the intersection of the lines of action and of centres. Evidently the distance between the cylinders does not affect the speed ratio since the base cylinder diameters are fixed.
- A pinion tooth touches a wheel tooth at the contact point C (the knot) which moves up the line of action and along the teeth faces as rotation proceeds.

- Since contact cannot occur outside the teeth, it takes place along the line of action only between the points Q_2 and Q_1 on the line of action and inside both addendum circles. The line segment $Q_2 Q_1$ is named the ***path of contact.***

PATH OF CONTACT:

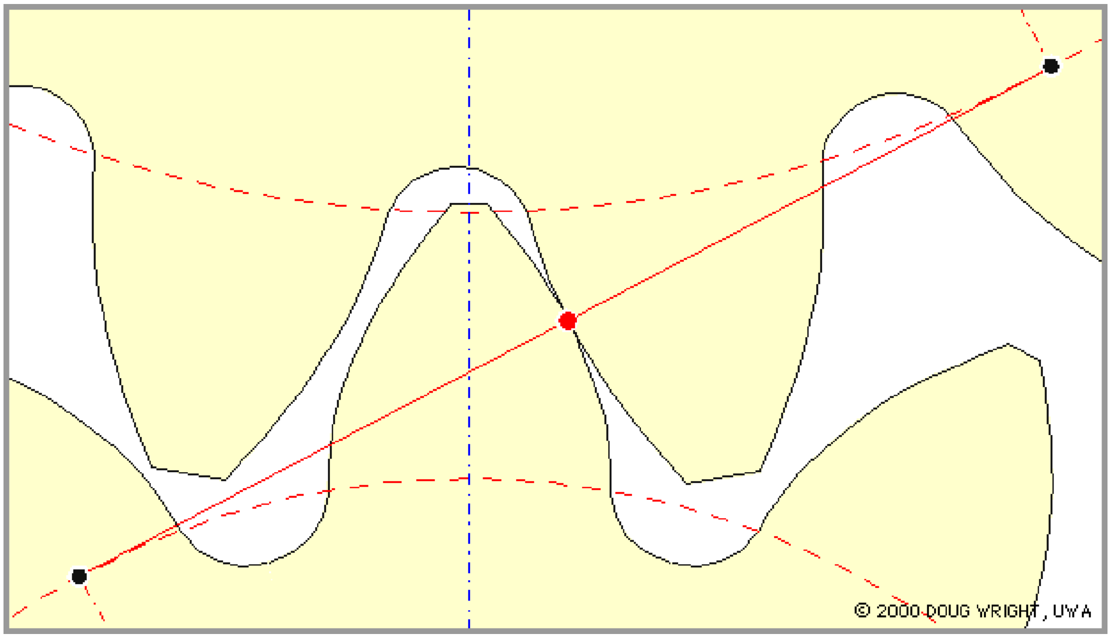


Fig : 9 The path of contact

The Figure shows clearly :

- the contact point marching along the line of action
- the path of contact bounded by the two addenda
- the orthogonality between line of action and involute tooth flanks at the contact point
- how load is transferred from one pair of contacting teeth to the next as rotation proceeds
- relative sliding between the teeth - particularly noticable at the beginning and end of contact
- guaranteed tooth tip clearance due to the dedendum exceeding the addendum
- a significant gap between the non-drive face of a pinion tooth and the adjacent wheel tooth

- The gap between the non-drive face of the pinion tooth and the adjacent wheel tooth is known as ***backlash.*** If the rotational sense of the pinion were to reverse, then a period of unrestrained pinion motion would take place until the backlash gap closed and contact with the wheel tooth re-established impulsively.

- Shock in a torsionally vibrating drive is exacerbated by significant backlash, though a small amount of backlash is provided in all drives to prevent binding due to manufacturing or mounting inaccuracies and to facilitate lubrication.
- Backlash may be reduced by subtle alterations to tooth profile or by shortening the centre distance from the extended value, however we consider gears meshing only at the extended centre distance.
- The average number of teeth in contact is an important parameter - if it is too low due to the use of inappropriate profile shifts or to an excessive centre distance for example, then manufacturing inaccuracies may lead to loss of kinematic continuity - that is to impact, vibration and noise.

The average number of teeth in contact is also a guide to load sharing between teeth; it is termed the ***contact ratio,*** ε_γ, given by :-

ε_γ = length of path of contact / distance between teeth along the line of action

= $Q_2 PQ_1$ / base pitch, p_o and for extended centres with for the 20^o system :

8) $(2\pi \cos\alpha) \varepsilon_\gamma = \Sigma_{i = 1,2} \sqrt{[(z_i + 2(1+s_i))^2 - (z_i \cos\alpha)^2]} - \sqrt{[(\Sigma z + 2 \Sigma s)^2 - (\Sigma z \cos\alpha)^2]}$

- Gears having a contact ratio below about 1.2 are not normally recommended as the gears themselves, their shafts and bearings would all require especial care in design and manufacture to preserve conjugacy.
- A pinion tooth touches a wheel tooth at the contact point C (the knot) which moves up the line of action and along the teeth faces as rotation proceeds.
- Since contact cannot occur outside the teeth, it takes place along the line of action only between the points Q_2 and Q_1 on the line of action and inside both addendum circles. The line segment $Q_2 Q_1$ is named the ***path of contact.***

BACK LASH

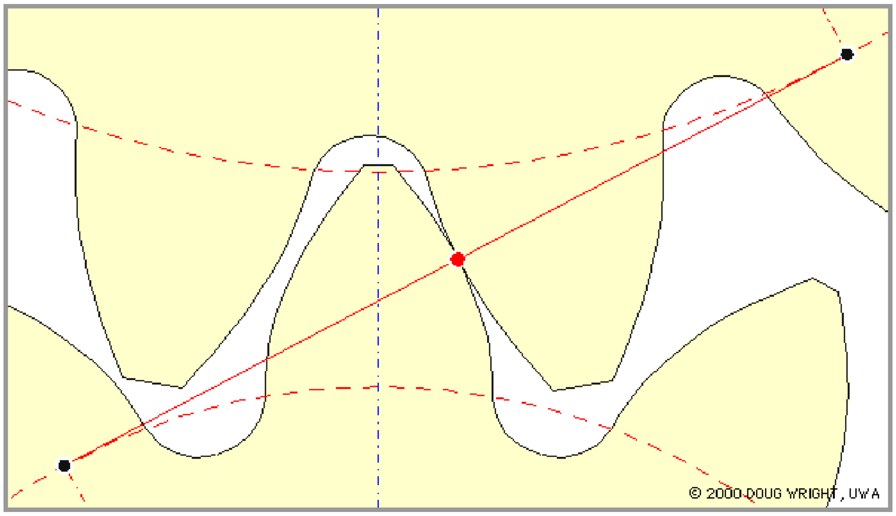


Fig : 10. The path of contact (back lash)

The Figure shows clearly :

- the contact point marching along the line of action
- the path of contact bounded by the two addenda
- the orthogonality between line of action and involute tooth flanks at the contact point
- how load is transferred from one pair of contacting teeth to the next as rotation proceeds
- relative sliding between the teeth - particularly noticable at the beginning and end of contact
- guaranteed tooth tip clearance due to the dedendum exceeding the addendum
- a significant gap between the non-drive face of a pinion tooth and the adjacent wheel tooth

v The gap between the non-drive face of the pinion tooth and the adjacent wheel tooth is known as ***backlash.*** If the rotational sense of the pinion were to reverse, then a period of unrestrained pinion motion would take place until the backlash gap closed and contact with the wheel tooth re-established impulsively.

v Backlash may be reduced by subtle alterations to tooth profile or by shortening the centre distance from the extended value, however we consider gears meshing only at the extended centre distance.

GEAR TRAIN

SIMPLE GEAR TRAIN:

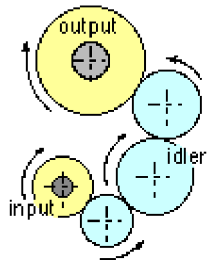


Fig. 11 .Simple Gear Train

- The only way that the input and output shafts of a gear pair can be made to rotate in the same sense is by interposition of an odd number of intermediate gears as shown in Fig 11 - these do not affect the speed ratio between input and output shafts.
- Such a gear train is called a ***simple*** train. If there is no power flow through the shaft of an intermediate gear then it is an ***idler*** gear.

COMPOUND GEAR TRAIN:

- A gear train comprising two or more pairs is termed ***compound*** when the wheel of one stage is mounted on the same shaft as the pinion of the next stage.
- A compound train as in the above gearbox is used when the desired speed ratio cannot be achieved economically by a single pair.
- Applying (**1**) to each stage in turn, the overall speed ratio for a compound train is found to be the product of the speed ratios for the individual stages.

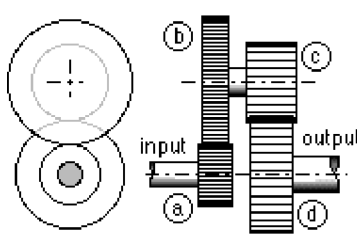


Fig. 12. Compound Gear Train

- Selecting suitable integral tooth numbers to provide a specified speed ratio can be awkward if the speed tolerance is tight and the range of available tooth numbers is limited.

v Unlike the above gearbox, the input and output shafts are coaxial in the train illustrated here; this is rather an unusual feature, but necessary in certain change speed boxes and the like.

In the next section we look at a particular gear train arrangement called an **epicyclic gear train**, before focusing on details of **gear tooth shape** and manufacture.

EPICYCLIC GEAR TRAINS

An epicyclic train is often suitable when a large torque/speed ratio is required in a compact envelope. It is made up of a number of ***elements*** which are interconnected to form the train. Each element consists of the three ***components*** illustrated below :

- A ***central gear*** (**c**) which rotates at angular velocity ω_c about the fixed axis O-O of the element, under the action of the torque T_c applied to the central gear's integral shaft; this central gear may be either an **external** gear (also referred to as a *sun* gear) Fig 13.a(1a), or an **internal** gear, Fig 13.a (1b).
- An ***arm*** (**a**) which rotates at angular velocity ω_a about the same O-O axis under the action of the torque, T_a - an axle A rigidly attached to the end of the arm carries
- ***A planet gear*** (**p**) which rotates freely on the axle A at angular velocity ω_p, meshing with the central gear at the pitch point P - the torque T_p acts on the planet gear itself, not on its axle, A.

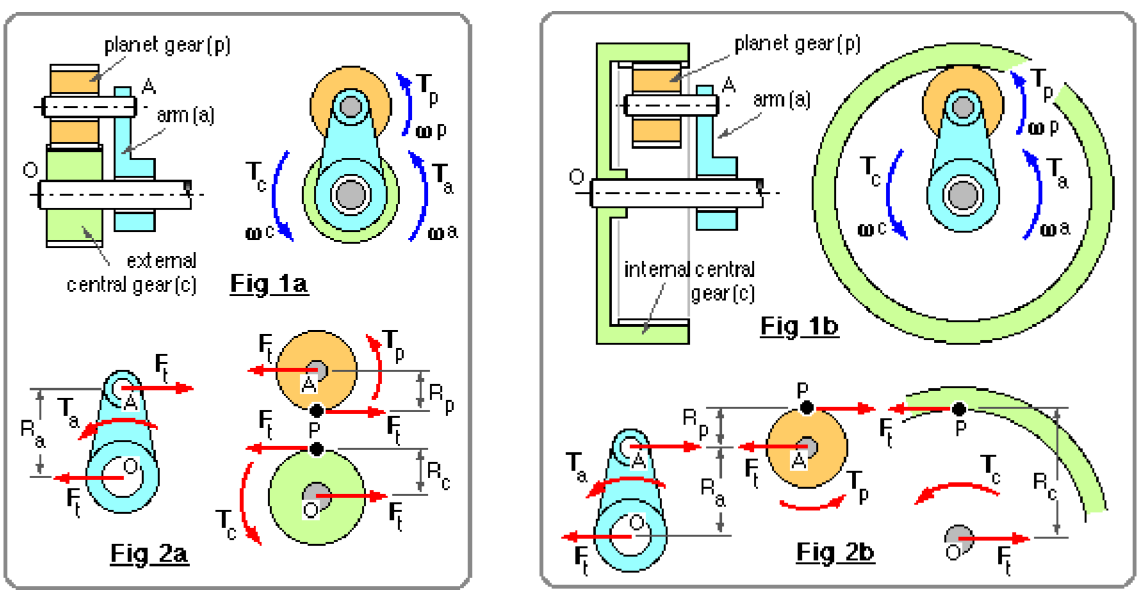


Fig 13.a : **Epicyclic gear trains**

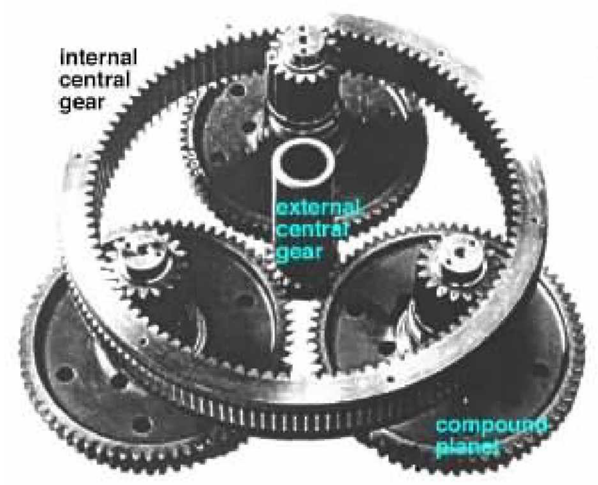


Fig 13.b : **Epicyclic gear trains**

- The epicyclic gear photographed here without its arms consists of two elements. The central gear of one element is an exter+nal gear; the central gear of the other element is an internal gear.
- The three identical planets of one element are compounded with (joined to) those of the second element.
- We shall examine first the angular velocities and torques in a single three-component element as they relate to the tooth numbers of central and planet gears, z_c and z_p respectively.
- The kinetic relations for a complete epicyclic train consisting of two or more elements may then be deduced easily by combining appropriately the relations for the individual elements.
- All angular velocities, ω, are absolute and constant, and the torques, T, are external to the three-component element; for convenience all these variables are taken positive in one particular sense, say anticlockwise as here. Friction is presumed negligible, ie. the system is ideal.

There are two contacts between the components :

- the planet engages with the central gear at the pitch point P where the action / reaction due to tooth contact is the tangential force F_t, the radial component being irrelevant;

- the free rotary contact between planet gear and axle A requires a radial force action / reaction; the magnitude of this force at A must also be F_t as sketched, for equilibrium of the planet.

With velocities taken to be positive leftwards for example, we have for the external central gear

- geometry from Fig 13.a (2a) : $R_a = R_c + R_p$
- velocity of P : $v_P = v_A + v_{PA}$

  so with the given senses : $\omega_c R_c = \omega_a R_a - \omega_p R_p$

- torques from Fig 13.a (2a) : $F_t = -T_c / R_c = -T_p / R_p = T_a / R_a$

For the internal central gear :

- geometry from Fig 13.a (2a): $R_a = R_c - R_p$
- velocity of P : $v_P = v_A + v_{PA}$ so with the given senses : $\omega_c R_c = \omega_a R_a + \omega_p R_p$
- torques from Fig 13.a (2a): $F_t = -T_c / R_c = T_p / R_p = T_a / R_a$

Substituting for R_a from the geometric equations into the respective velocity and torque equations, and noting that $R_c/R_p = z_c/z_p$, leads to the same result for both internal and external central gear arrangements.

These are the desired relations for the three-component element :

$$(\omega_c - \omega_a)\, z_c + (\omega_p - \omega_a)\, z_p = 0 \, ;$$
$$T_c / z_c = T_p / z_p = -T_a / (z_c + z_p) \, .$$

In which z_c is taken to be a positive integer for an external central gear, and a negative integer for an internal central gear.

- It is apparent that the element has **one** degree of kinetic (torque) freedom since only one of the three torques may be arbitrarily defined, the other two following from the two equations.
- On the other hand the element possesses **two** degrees of kinematic freedom, as any two of the three velocities may be arbitrarily chosen, the third being dictated by the single equation.

From the net external torque on the three-component element as a whole is :

$$\Sigma T \;=\; T_c + T_p + T_a \;=\; T_c \{ 1 + z_p / z_c - (z_c + z_p)/z_c \} \;=\; 0$$

which indicates that equilibrium of the element is assured.

Energy is supplied to the element through any component whose torque and velocity senses are identical. From (**2**) the total external power being fed into the three-component element is

$$\Sigma P \;=\; P_c + P_p + P_a \;=\; \omega_c T_c + \omega_p T_p + \omega_a T_a \;=\; T_c \{ \omega_c + \omega_p z_p / z_c - \omega_a (z_c + z_p)/z_c \}$$

$$= \; T_c \{ (\omega_c - \omega_a) z_c + (\omega_p - \omega_a) z_p \} / z_c \;=\; 0$$

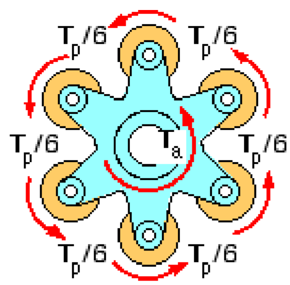


Fig 14: planet carrier.

- In practice, a number of identical planets are employed for balance and shaft load minimisation.
- Since (**2**) deal only with effects external to the element, this multiplicity of planets is analytically irrelevant provided T_p is interpreted as being the total torque on all the planets, which is shared equally between them as suggested by the sketch here.
- The reason for the *sun- and- planet* terminology is obvious; the arm is often referred to as the ***spider*** or ***planet carrier.***
- Application of the element relations to a complete train is carried out as shown in the example which follows.

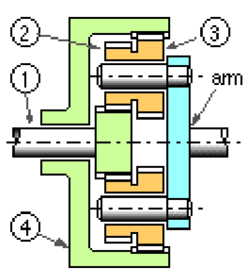


Fig 15: sun and wheel

- An epicyclic train consists of two three-component elements of the kind examined above. The first element comprises the external sun gear 1 and planet 2; the second comprises the planet 3 and internal ring gear 4.

- The planets 2 and 3 are compounded together on the common arm axles. Determine the relationships between the kinetic variables external to the train in terms of the tooth numbers z_1, z_2, z_3 & z_4.

- The train is analysed via equations (**2**) applied to the two elements in turn, together with the appropriate equations which set out the velocity and torque constraints across the interface between the two elements **1-2-arm** and **3-4-arm**.

UNIT V

FRICTION

SYLLUBUS

"Surface contacts-Sliding and Rolling friction - Friction drives – Friction in screw threads - Friction clutches - Belt and rope drives, Friction aspects in Brakes – Friction in vehicle propulsion and braking".

INTRODUCTION

- When a body slides (rolls) or made to slide (roll) relative to a second body, with which it is in contact, there is a resistance to the relative motion. The resistance so encountered is called friction.
- The force resisting relative motion is called force of friction. Force of friction acts in a direction opposite to that of relative motion and is tangential to the contacting surfaces of the two bodies in contact.
- At every joint in a machine, there is a loss of energy owing to friction. A proper understanding about friction as a phenomenon enables us to reduce frictional forces. In a number of applications, on the other hand, friction is considered to be quite useful.
- Friction drives like belt and rope drives, friction clutches, variable speed drives are some of applications of this type.

TYPES OF FRICTION

Dry Friction

- This type of friction exists between two bodies having relative motion and whose contacting surfaces are dry and not separated by any lubricant.
- It is further subdivided in two types as Sliding friction and rolling friction. Sliding friction is the friction in which the contacting surfaces have a sliding motion relative to each other.
- Rolling friction is the friction between two bodies in contact when they have a relative motion of pure rolling.

Skin or Greasy Friction

- When contact surfaces of two bodies, in relative motion, are separated by a film of lubricant of small thickness, skin or greasy friction is said to exist between them.
- This type of friction is also known as boundary friction.

Film or Viscous Friction

- When contacting surfaces of two bodies, in relative motion, are completely separated by a relatively thick film of fluid, viscous friction is said to exist between the two.

Limiting Friction

- The maximum value of frictional force, which comes into play when one body slides or tends to slide over another body, is known as Limiting Friction.

Laws of Friction

1. Force of friction always acts in a direction in which the body tends to move.
2. Force of friction is directly proportional to the normal load between the surfaces for a given pair of materials.
3. The force of friction depends upon the materials of the contacting surfaces.
4. The force of friction is independent of the area of contact surfaces for a given normal load.

Coefficient of Friction (μ)

It is defined as the ratio between the limiting friction (F) and the normal reaction(R).

$$\mu = F / R$$

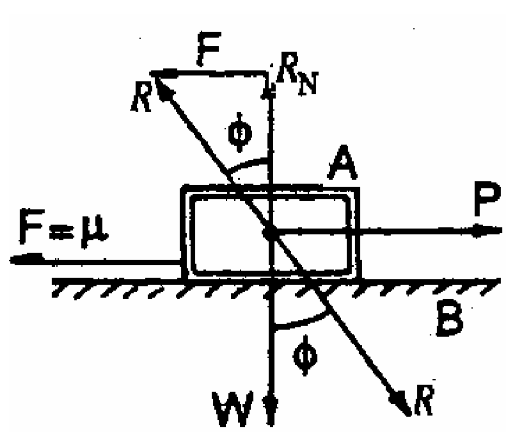


Figure. 1. Angle of friction (θ)

Let R is the resultant of normal reaction (R_N) and the limiting friction (F). Then the angle between R and R_N is known as the angle of friction.

$$\tan\theta = F / R. = \mu$$

Angle of Repose

- Consider that a body of weight (W) resting on an inclined plane. If the angle of inclination of the plane to the horizontal is such that the body begins to move down the plane, then the inclination of the plane is known as angle of repose.
- The angle of repose is equal to angle of friction.

INCLINED PLANE

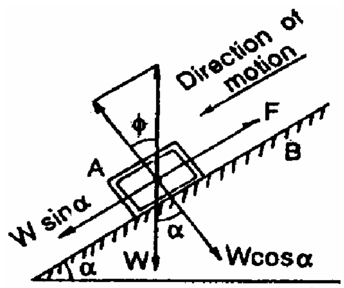


Figure.2. Inclined Plane

Body at Rest

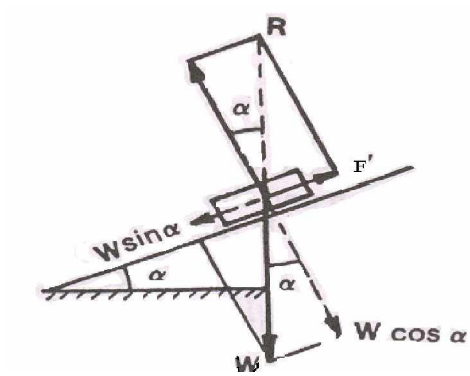


Figure.3. Body at Rest

When a body is at rest on an inclined plane making an angle α with the horizontal, the forces acting on the body are (Figure 3)

Let W = weight of body

R_N, = Normal reaction

F' = force resisting the motion of body.

From equilibrium conditions, W$\sin\alpha$ = F' and W$\cos\alpha = R_N$.

If the angle of inclination of plane is increased, the body will just slide down the plane of its own when

$$W \sin\alpha = F' = \mu R_N = \mu W \cos\alpha$$

$$\tan\alpha = \mu = \tan\phi \quad \text{(or)} \quad \alpha = \phi$$

This maximum value of angle of inclination of plane with the horizontal when the body starts sliding of its own is known as the angle of repose or limiting angle of friction.

Motion up the Plane

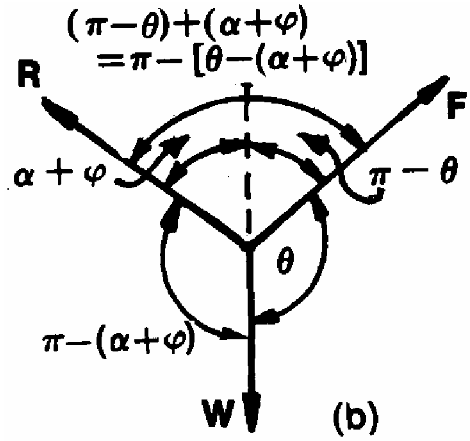


Figure 4: Motion up the plane

Consider a body moving up an inclined plane under the action of a force F as shown in Figure 4. Applying conditions of equilibrium and solving the equations obtained, we get the minimum force required to be applied, for equilibrium condition as $F_{min} = W \sin(\alpha + \psi)$

Efficiency:

The efficiency of an inclined plane, when a body slides up the plane, is defined as the ratio of the forces required to move the body without consideration and with consideration of force of friction. From the analysis the expression for the efficiency is found to be

$$\eta = \frac{\cot(\alpha + \theta) - \cot\theta}{\cot\alpha - \cot\theta}$$

Motion down the Plane

When the body moves down the plane, the force of friction F' (= μ R_n) acts in the upwards direction and the reaction **R,** i.e. the combination of R_n and **F'** is inclined backwards. Applying conditions of equilibrium we get the minimum force, required to be applied as

$F_{min} = W \sin(\psi - \alpha)$

Efficiency:

"Efficiency of the inclined plane when the body slides down the plane is defined as the ratio of the forces required to move the body with and without the consideration of force of friction".

$$\eta = \frac{\cot\alpha - \cot\theta}{\cot(\varphi - \alpha) + \cot\theta}$$

Square Threads

- A square threaded screw used as a jack to raise a load W.
- Faces of the square threads in the sectional vies are normal to the axis of the spindle.
- Force F acting horizontally is the force at the screw thread required to slide the load W up the inclined plane.

$$F = W\frac{1+\mu\pi d}{\pi d - \mu l} \qquad F = \frac{W\sin(\alpha+\varphi)}{\sin[90° - (\alpha+\varphi)]}$$

The force F required to be applied is given by

Substituting tanα = l / πd and tanϕ = μ and simplifying we get

A bar is, usually fixed to the screw head to use as a lever for the application of force.

Let f = force applied at the end of the bar of length L

Then

$$f L = F (d/2) = F\times r \qquad \text{or} \qquad f = Fr/L = W \times r / L\ [\tan(\alpha + \phi)]$$

If the weight is lowered, the expressions for F and f are given by

$$f = \frac{Wr}{L}\tan(\varphi-\alpha)$$

$$F = \frac{W\sin(\varphi-\alpha)}{\sin(90° + (\varphi - \alpha))}$$

$$\text{Screw efficiency} = \frac{\text{work done in lifting the load/rev}}{\text{work done by the applied force/rev}}$$

Screw efficiency η is defined as

$$\eta = \frac{W\times l}{F\times \pi d} = \frac{W}{F}\times\frac{l}{\pi d}$$

Therefore the above equation can be obtained in terms of α and ϕ as

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\varphi)}$$

The efficiency η is maximum when dη/dα = 0 giving the necessary condition for maximum efficiency as $\alpha = 45^0 - \phi/2$

V-THREADS

$$\text{Friction force on the surface} = \mu R_n = \mu \frac{W}{\cos\beta} = \frac{\mu}{\cos\beta} W = \mu' W$$

- In this case the faces are inclined to the axis of spindle. Figure shows a section of V-thread in which 2β is the angle between the faces of the thread.

- If R_N is the normal reaction, then the axial component of R_n must be equal to W i.e. $W = R_n \cos\beta$

- This shows that the coefficient of friction μ (or $\tan\phi$) as used in relations for the square threads is to be replaced by μ' or $\mu/\cos\beta$ or $\tan\psi/\cos\beta$ to adapt them to V-thread

PIVOTS AND COLLARS

- When a rotating shaft is subjected to an axial load, the thrust (axial force) is taken either by a pivot or a collar.
- Examples are the shaft of a steam turbine, propeller shaft of a ship etc.

COLLAR BEARING

A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.

The surface of the collar may be plane normal to the shaft or of conical shape.

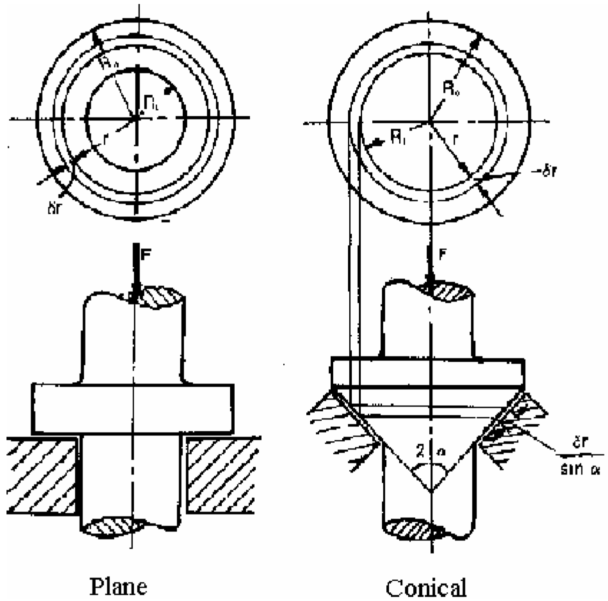


Figure 5: Collar Bearing

PIVOT BEARING

When the axial load is taken by the end of the shaft, which is inserted in a recess to bear the thrust, Figure 6.

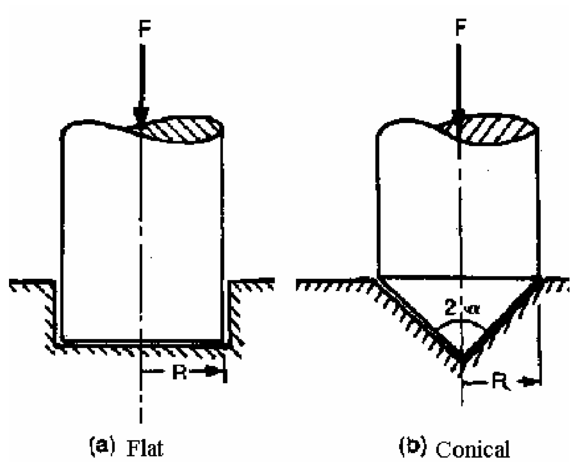


Figure 6: Pivot Bearing

It is called a pivot bearing or simply a pivot. It is also known as footstep bearing.

Friction torque of a collar bearing or pivot bearing is calculated on the basis of following two assumptions:

- Uniform Pressure theory
- Uniform Wear theory

Each assumption leads to a different value of torque.

Uniform Pressure theory

In this case the intensity of pressure on the bearing surface is assumed to be constant and the intensity of pressure is given by

$$\text{Pressure} = \frac{\text{axial force}}{\text{cross-sectional area}}$$

$$p = \frac{F}{\pi(R_o^2 - R_i^2)}$$

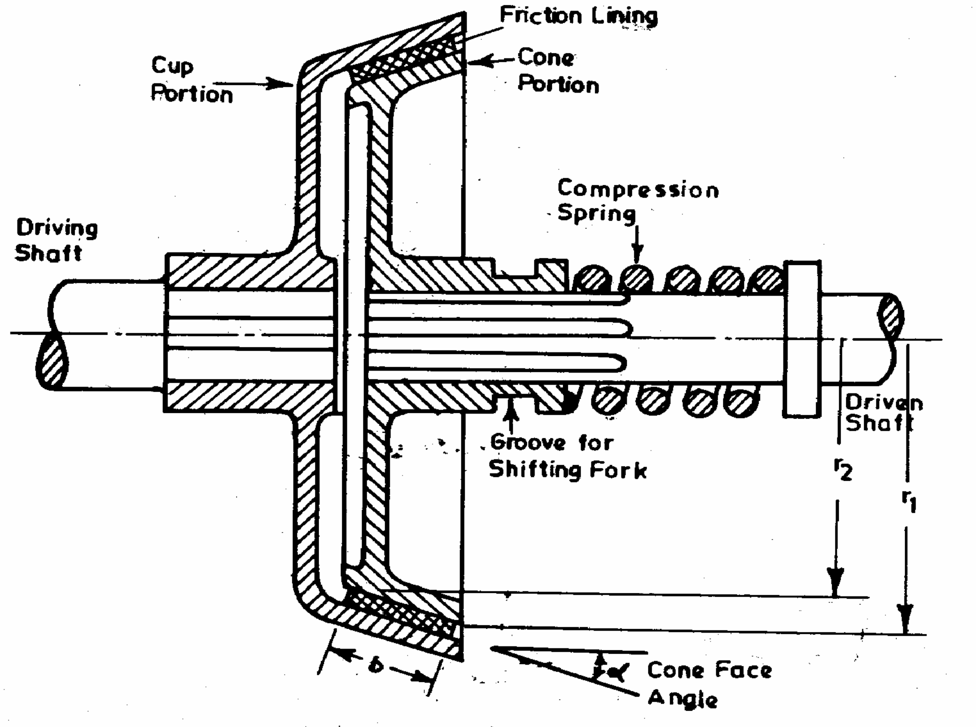
Where R_o is the outer radius of the collar and R_i is the inner radius of the collar.

Uniform Wear theory

In this case wearing of the bearing surface is assumed to be uniform. Under this assumption

$P_o\, r_o = P_i\, r_i = P.\, r$ = constant for uniform rate of wear

CONE CLUTCH



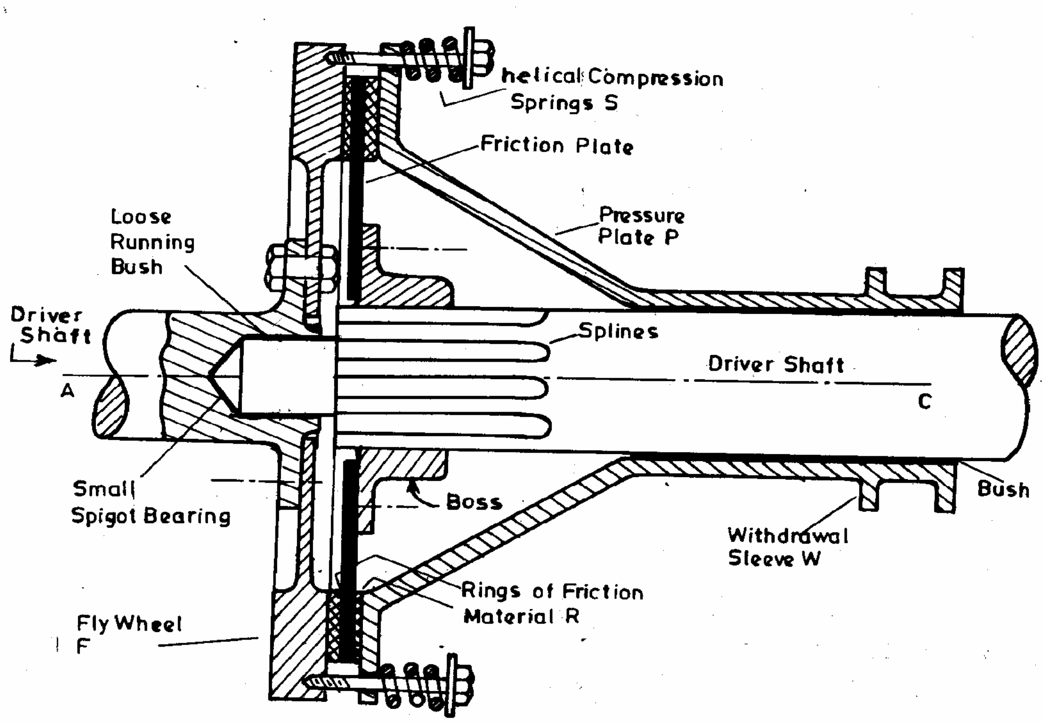


Fig 7: Cone Clutch

Thus, let

W = weight of each shoe

n = number of shoes

r = radial distance of c.g. of shoe from the centre of the spider.

R = Inside rim–radius for pulley,

N = running speed of pulley in r.p.m.

ω = Angular speed of pulley corresponding to the r.p.m. of N.

ω_1 = Angular speed at which the engagement occurs,

and, μ = coefficient of friction between shoe and rim ;

When rotating at a speed ω ($\omega > \omega_1$), the centrifugal force F_c and spring force acting on each shoe are :

$$F_c = \frac{W}{g} \cdot r \cdot \omega^2$$

The clearance c between shoe and rim being small, r can be assumed to remain constant)

and $$P = \frac{W}{g} \cdot r \cdot \omega_1^2$$

Thus, the nett outward force, pressing the shoe against living is therefore

$$= (F_c - P)$$

$$= \left(\frac{W}{g} \cdot r \cdot \omega^2 - \frac{W}{g} \cdot r \cdot \omega_1^2\right) = \frac{W}{g} \cdot (r) \cdot (\omega^2 - \omega_1^2) \quad \text{... a}$$

Therefore, the frictional force acting on each shoe,

$$= \mu\,(F_c - P) = \mu\left(\frac{W}{g} \cdot r\right)(\omega^2 - \omega_1^2) \quad \text{... b}$$

The frictional torque transmitted by n such shoes,

$$T_f = n\left(\mu \cdot \frac{W}{g} \cdot r\right) \cdot R \cdot (\omega^2 - \omega_1^2) \quad \text{... c}$$

Thus, if frictional torque T_f, n, μ , r and R are known, the weight W can be obtained for given values of ω and ω_1.

To evaluate size of the shoe, let

l = Arcular length of contact of shoe surface

b = face width of shoes.

R = contact radius of shoes and equals inside rim radius,

θ = Angle subtended at the centre of spider by shoe

p = intensity of shoe–pressure on rim. (For reasonable shoe life this may be restricted to 9·8 N/cm^2)

Since, l $= R \cdot \theta$ and, area of contact for each shoe $= l.b$, the force with which shoe is pressed against rim is,

$$= p \cdot l \cdot b = p \cdot (R \cdot \theta) \cdot b$$

BELT AND ROPE DRIVE

INTRODUCTION

- The velocity of the belt.
- The tension under which the belt is placed on the pulleys.
- The arc of contact between the belt and the smaller pulley
- The conditions under which the belt is used.

It may be noted that

1. The shafts should be properly in line to insure uniform tension across the belt section.
2. The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.
3. The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.
4. A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt
5. The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.
6. In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 meters and the minimum should not be less than 3.5 times the diameter of the larger pulley.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

Light drives:

These are used to transmit small powers at belt speeds up to about 10 m is, as in agricultural machines and small machine tools.

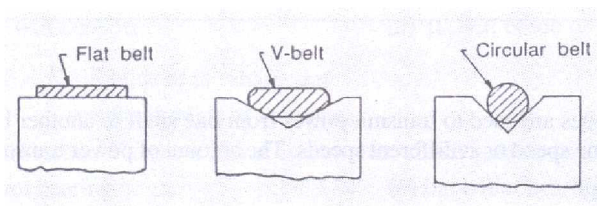
Medium drives:

These are used to transmit medium power at belt speeds over 10 m but up to 22 m as in machine tools.

Heavy drives:

These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

Types of Belts



(a) Flat belt. (b) V-belt. (c) Circular belt.

Fig.7.Types of belts.

Though there are many types of belts used these days, yet the following are important from the subject point of view:

Flat belt:

The flat belt, as shown in Fig. 7 (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 meters apart.

V-belt:

The V-belt, as shown in Fig. 7(b), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

Circular belt or rope:

- The circular belt or rope, as shown in Fig. 7(c), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.
- If a huge amount of power is to be transmitted, then a single belt may not be sufficient.
- In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used.
- Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.